

Factors

Factor : Multiplying two whole numbers gives a product. The numbers that we multiply are the factors of the product.

Example

$$\begin{array}{c}
 2 \times 3 = 6 \\
 \swarrow \quad \searrow \\
 \text{Factors} \quad \text{Product}
 \end{array}$$

2, 3 are factors of 6

Example $15 = 3 \times 5$ therefore, 3 and 5 are the factors of 15.

Also $15 = 1 \times 15$

Therefore, 1, 15 are factors of 15.

A factor divides a number completely without leaving any remainder.

Example Find factors of 6.

$$\begin{array}{r}
 6 \\
 1 \overline{)6} \\
 \hline
 -6 \\
 \hline
 0
 \end{array}
 \quad
 \begin{array}{r}
 3 \\
 2 \overline{)6} \\
 \hline
 -6 \\
 \hline
 0
 \end{array}
 \quad
 \begin{array}{r}
 2 \\
 3 \overline{)6} \\
 \hline
 -6 \\
 \hline
 0
 \end{array}
 \quad
 \begin{array}{r}
 1 \\
 6 \overline{)6} \\
 \hline
 -6 \\
 \hline
 0
 \end{array}$$

Factors of 6 are 1, 2, 3 and 6.

The number 1 is the smallest factor of every numbers.

Every number will have a minimum of two factors, 1 and the number itself.

Multiples: Multiples of a number are the numbers obtained by multiplying a number by a whole number.

Example Multiples of 3 are

$0 \times 3 = 0$, 0 is a multiple of 3

$1 \times 3 = 3$, 3 is a multiple of 3

$2 \times 3 = 6$, 6 is a multiple of 3

$3 \times 3 = 9$, 9 is a multiple of 3 and so on.

Factors of 3 are 1 and 3.

Example Multiples of 6 are

$$\left. \begin{array}{l}
 0 \times 6 = 0 \\
 1 \times 6 = 6 \\
 2 \times 6 = 12 \\
 3 \times 6 = 18 \\
 \text{as so on.}
 \end{array} \right\} \begin{array}{l}
 \text{These are multiples} \\
 \text{of 6}
 \end{array}$$

Note: There are negative factors and multiples as well.

Example

$-1 \times -12 = 12$

$-2 \times -6 = 12$

$-3 \times -4 = 12$

So all the factors of 12 are 1, 2, 3, 4, 6 and 12 and -1, -2, -3, -4, -6 and -12.

1, 2, 3, 4, 6 and 12 are positive factors of 12 and -1, -2, -3, -4, -6 and -12 are negative factors of 12. Similarly negative multiples of 12 are

$$\left. \begin{array}{l}
 -1 \times -12 = -12 \\
 -2 \times -12 = -24 \\
 -3 \times -12 = -36
 \end{array} \right\} \begin{array}{l}
 \text{Negative multiples} \\
 \text{of 12}
 \end{array}$$

and so on.

Work sheet 1.1

- 1 Besides 1 and 6 which one is factor of 6? _____
- 2 Which number is a factor of 11?
(a) 3 (b) 7 (c) 5 (c) 11

Name: _____
Grade: _____

1.1

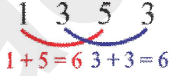
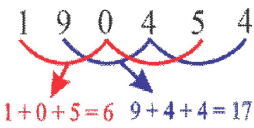
- 3 Which is factor of 9?
(a) 5 (b) 3 (c) 2 (d) 6 _____
- 4 Which numbers are the factors of 15 ?
(a) 5 (b) 3 (c) 1 (d) 15 _____
- 5 Which is a factor of every number?
(a) 0 (b) 1 (c) 2 (d) 3 _____
- 6 Is 6 a factor of 78? Yes or No _____
- 7 What are the factors of 24?
(a) 2, 6 (b) 2, 4, 3, 8, 12, 1, 24
(c) 1, 4, 8, 12 (d) None of the above _____
- 8 Pick the odd one out.
(a) 2 (b) 4 (c) 6 (d) 7 _____
- 9 Which number is a factor of 14?
(a) 7 (b) 6 (c) 8 (d) 9 _____
- 10 Write all the factors of 12. _____
- 11 Write first 5 positive multiples of 11. _____
- 12 Write 4 negative multiples of 15. _____
- 13 Which number has exactly one factor? _____
- 14 Is there any counting number having no factor at all? _____

- 15 Find numbers between 1 and 100 having exactly three factors. _____
- 16 The largest factor of a number is _____
- 17 The first multiple of a number is _____
- 18 All multiples of 9 are multiples of _____ also. _____
- 19 Write first three positive multiples of 8. _____
- 20 Write the multiples of 6 which are greater than 20 and less than 50. _____
- 21 Write all the multiples of 20 which are less than 200. _____
- 22 How many factor pairs are there for the number 28? _____
- 23 28, 35, 42 and 49 are multiple of which number? _____
- 24 Is 13 is a factor of 156? _____
- 25 Is 187 is a multiple of 17? _____
- 26 Write all the factors of 91. _____
- 27 What is ninth positive multiple of 19? _____
- 28 Write 5 negative multiples of 9. _____
- 29 Which number is ninth multiple of 14? _____

Divisibility test

By learning and practising divisibility tests, we can easily determine whether a given integer is divisible by the given number or not. 'Divisible' by means when you divide a number by another number, the result is a whole number.

Divisibility by	Rule	Example
1	Every number is divisible by 1.	5 is divisible by 1. 3,700 is divisible by 1.
2	If a number is even or a number whose digit in ones place is an even number i.e. 2,4,6,8, or 0 is always divisible by 2 .	5,026 is divisible by 2 because 6 is at ones place and is even number. 127 is not divisible by 2 because 7 which is at ones place is odd number.
3	A number is completely divisible by 3, if the sum of its digits is divisible by 3 or a multiple of 3.	To check whether 708 is divisible by 3 or not, take sum of the digits (i.e. $7 + 0 + 8 = 15$). Clearly 15 is divisible by 3. Similarly, 519 is divisible by 3 and 311 is not divisible by 3.
4	If the last two (ones place and tens place) digits of a number are divisible by 4, then number is also divisible by 4.	3,020 is divisible by 4 as its last two digits (i.e. 20) are divisible by 4.
5	Numbers which has 0 or 5 in ones place are always divisible by 5.	10, 1000, 105, 1255, 2705 are divisible by 5 as 0 or 5 is in ones place.
6	A number which is divisible by both 2 and 3 is divisible by 6. So, if the number is even and sum of its digits is divisible by 3 then the given number is divisible by 6.	732 is divisible by 2 as it is an even number and sum of its digits (i.e. $7 + 3 + 2 = 12$) is also divisible by 3. Hence 732 is divisible by 6.
7	<p>The rule for divisibility by 7 :</p> <p>Step I : First double the digit which is at ones place.</p> <p>Step II: Subtract from remaining number. If the number is 0 or 2 digit number multiple of 7 then number is divisible by 7 or repeat the above steps.</p> <pre> graph TD A[2 x digit at ones place.] --> B[Subtract from remaining number.] B --> C{Is the number 0 or 2 digit number multiple of 7?} C -- yes --> D[Divisible by 7.] C -- No --> A </pre>	<p>1) 1,074 is not divisible by 7 because</p> <p>Step I: First take 4 which is in ones place and double it i.e. $2 \times 4 = 8$.</p> <p>Step II : Subtract from remaining number i.e. $107 - 8 = 99$ which is not a multiple of 7.</p> <p>2) Consider 1,211</p> <p>Step I: First take the digit which is in ones place and double it (i.e. $2 \times 1 = 2$).</p> <p>Step II: Subtract 2 from remaining number i.e. $121 - 2 = 119$</p> <p>Now take again 9 which is in ones place and then double it, $9 \times 2 = 18$ and subtract 18 from remaining. $11 - 18 = -7$ which is clearly divisible by 7, so 1,211 is divisible by 7.</p>

8	If the last three digits of a number are divisible by 8 then the number is divisible by 8.	7,856 is divisible by 8 because last three digits (i.e 856) are divisible by 8. It is easy to check weather 856 is divisible by 8 or not. We know that $8 = 2 \times 2 \times 2$. Divide 856 by 2 then again divide it by 2 and again by 2 to get the result.
9	If the sum of digits of the number is divisible by 9, then the number itself is divisible by 9.	5,058 is divisible by 9 because sum of its digits ($5 + 0 + 5 + 8 = 18$) is divisible by 9.
10	If the digit at ones place is 0 then number is divisible by 10.	1,050, 10,000 and 104,040 are divisible by 10 because digit in ones place is 0.
11	If the difference of the sum of alternative digits of a number is divisible by 11, then the number is also divisible by 11.	<p>a) 1353 is divisible by 11 because difference of sum of alternative digits is divisible by 11.</p>  <p>Difference is $6 - 6 = 0$ and clearly 0 is divisible by 11.</p> <p>b) Consider 190,454 Sum of alternative digits are</p>  <p>Difference is $17 - 6 = 11$ which is divisible by 11. Hence 190,454 is divisible by 11.</p>
12	If a number is divisible by 3 and by 4 then number is divisible by 12. Or Subtract the digit at ones place from twice the rest if the result is divisible by 12 then the number is also divisible by 12.	1116 is divisible by 12 because 1116 is divisible by 3 as well as by 4.
13	Add four times of the ones place digit to the remaining number and repeat the process until you get a two digit number. Now check if the two digit number is divisible by 13 or not. If it is divisible, then the given number is divisible by 13.	Consider 1,183. Add four times of the ones place (i.e. $3 \times 4 = 12$) to the remaining number (i.e. $118 + 12 = 130$). Clearly 130 is divisible by 13 so number 1,183 is also divisible by 13.
14	It must be divisible by 2 and by 7.	1,386 is divisible by 2 as well as by 7. So It is divisible by 14.
15	It must be divisible by 3 and by 5.	1,095 is divisible by 15 because it is divisible by 3 as well as by 5.

Name: _____
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1.2

Work sheet 1.2

- 1 Is 34,562 divided by 2? _____
- 2 Is 60,925 divided by 3? _____
- 3 Is 4,089 divided by 5? _____
- 4 Is 16,489 divided by 2? _____
- 5 Is 38,905 divided by 10? _____
- 6 Is 28,784 divided by 8? _____
- 7 Is 780,050 divided by 13? _____
- 8 Is 900,003 divided by 13? _____
- 9 Is 9,605,605 divided by 6? _____
- 10 Is 9,089,080 divided by 10? _____
- 11 Is 572,283 divided by 9? _____
- 12 Is 9,809,800 divided by 14? _____
- 13 Is 23,888,885 divided by 11? _____
- 14 Is 6,005,001 divided by 7? _____
- 15 Is 6,030,024 divided by 12? _____

1.2

Name: _____
Grade: _____

16 Is 5,005,005 divided by 13? _____

17 Is 705,645 divided by 5? _____

18 Is 1,576,757 divided by 5? _____

19 Is 2,012,010 divided by 3? _____

20 Is 539,752 divided by 8? _____

21 Is 968,579 divided by 5? _____

22 Is 409,853 divided by 9? _____

23 Is 1,001,001 divided by 13? _____

24 Is 854,682 divided by 2? _____

25 Is 2,130,128 divided by 7? _____

26 Is 3,389,053 divided by 14? _____

27 Is 4,557,826 divided by 13? _____

28 Is 7,965,409 divided by 3? _____

29 Is 684,572 divided by 8? _____

30 Is 4,659,132 divided by 9? _____

31 Is 9,505,060 divided by 11? _____

Name: _____
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32 Is 1,564,986 divided by 14? _____

33 Is 3,965,089 divided by 15? _____

Fill in the blank with a digit that makes the given statement true.

34 14,3__6 is divisible by 4. _____

35 34,27__ is divisible by 5. _____

36 987,89__ is divisible by 10. _____

37 10,__04 is divisible by 3. _____

38 14,__48 is divisible by 6. _____

39 48,0__0 is divisible by 7. _____

40 2__,472 is divisible by 8. _____

41 5__8,760 is divisible by 15. _____

42 20,0__0 is divisible by 13. _____

43 __1,212 is divisible by 14. _____

44 80,94__ is divisible by 12. _____

45 28,1__6 is divisible by 9. _____

46 3,4__3,410 is divisible by 11. _____

Prime and composite

A prime is an integer greater than 1 which is divisible only by itself and 1. So 1 is not a prime.

Example 11 is a prime as it is only divisible by 1 and itself.

prime numbers between 1 and 50 are

2, 3, 5, 7, 11, 13, 17, 19, 23, 29

31, 37, 41, 43, 47.

2 is the only even prime number.

A composite number is a positive integer that can be formed by multiplying two smaller positive integers. Equivalently, it is a positive integer that has at least one divisor other than 1 and itself. So the composite numbers are exactly the numbers that are neither prime nor a unit.

Example 15 is a composite number because it is a product of the two smaller integers 5 and 3.

Example $120 = 2^3 \cdot 3 \cdot 5$
So, 120 is a composite number.

Work sheet 1.3

Find prime numbers lying between the following (end points included).

- 1 0 and 10 _____ 2 15 and 25 _____ 3 50 and 55 _____
4 0 and 12 _____ 5 30 and 70 _____

Express each integer as a product of prime numbers.

- 6 8 _____ 7 14 _____ 8 12 _____
9 21 _____ 10 111 _____ 11 86 _____

Write whether the number is prime or composite.

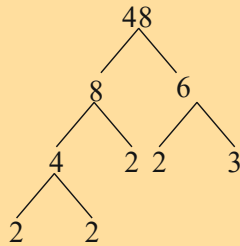
- 12 13 _____ 13 93 _____ 14 30 _____
15 23 _____ 16 19 _____ 17 36 _____
18 40 _____ 19 89 _____ 20 71 _____
21 59 _____ 22 80 _____ 23 37 _____
24 61 _____ 25 2 _____

Prime factorization

The method of prime factorization is used to express a given number (integer) as a product of prime numbers.

If a prime number occurs more than once in the factorization, it is usually expressed in exponential form to make it look more compact.

Example



$$48 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3.$$

Example $100 = 10 \times 10 = 5 \times 2 \times 5 \times 2 = 5^2 \times 2^2$

Example $950 = 2 \times 5 \times 5 \times 19$
 $= 2 \times 5^2 \times 19$

Work sheet 1.4

Write the prime factorization of the following.

1 10 _____

2 16 _____

3 24 _____

4 56 _____

5 81 _____

6 91 _____

7 125 _____

8 132 _____

9 66 _____

10 105 _____

11 500 _____

12 250 _____

13 180 _____

14 200 _____

15 1000 _____

16 405 _____

Greatest common factor (GCF)

The greatest common factor (GCF) of two integers is the greatest factor that divides both the integers. Simply we can say GCD of two integers is the largest integer dividing both.

Example Find GCF of 10 and 15.

$$10 = 2 \times 5$$

$$15 = 3 \times 5$$

Clearly 5 is common factor and greatest too.

So GCF of 10 and 15 is 5.

Example GCD of 4 and 12 is 4 because.

$$4 = 2 \times 2 = 2^2$$

$$12 = 6 \times 2 = 3 \times 2 \times 2 = 3 \times 2^2$$

Clearly 2^2 is common factor and greatest too.

To find GCF of two numbers, always start out the same way, you find the prime factorisation of the two numbers.

Example Find the GCF of 2940 and 3150.

First we will factorise the given numbers.

2	2940	2	3150
2	1470	3	1575
3	735	3	525
5	245	5	175
7	49	5	35
7		7	

We divided each of the given numbers by the smallest prime possible until we ended up with a prime number.

$$\text{So } 2940 = \boxed{2} \times 2 \times \boxed{3} \times \boxed{5} \times 7 \times \boxed{7}$$

$$3150 = \boxed{2} \times 3 \times \boxed{3} \times \boxed{5} \times 5 \times \boxed{7}$$

So GCF is $2 \times 3 \times 5 \times 7 = 210$

Example Let $a = 270$, $b = 252$. Find gcd of a and b .

First of all factor the given integers.

$$270 = 2^1 \cdot 3^3 \cdot 5^1$$

$$252 = 2^2 \cdot 3^2 \cdot 7^1$$

Now, match the prime divisors of two integers by inserting, any missing prime raised to the power zero if necessary.

$$270 = 2^1 \cdot 3^3 \cdot 5^1 \cdot 7^0$$

$$252 = 2^2 \cdot 3^2 \cdot 5^0 \cdot 7^1$$

Now, the GCD is the product of all above primes, each raised to the smallest of the two exponents.

$$\text{gcd}(a, b) = \text{gcd}(270, 252)$$

$$= 2^1 \cdot 3^2 \cdot 5^0 \cdot 7^0$$

$$= 2 \times 9 \times 1 \times 1 = 18$$

Work sheet 1.5

Compute the greatest common divisor of the following pairs of integers.

1 (1, 2) _____

2 (21, 18) _____

3 (55, 20) _____

4 (70, 84) _____

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Find GCD (a, b) where a and b are

5 $a = 4, b = 9, \text{gcd}(a, b)$ _____

6 $a = 108, b = 180, \text{gcd}(a, b)$ _____

7 $a = 140, b = 200, \text{gcd}(a, b)$ _____

8 $\text{gcd}(70, 84)$ _____
(a) 14 (b) 15 (c) 16 (d) 17

9 $\text{gcd}(-77, 60)$ _____
(a) 2 (b) 1 (c) 3 (d) 5

What is the GCD of the given numbers?

10 9 and 18 _____

11 14 and 7 _____

12 18 and 12 _____

13 20 and 12 _____

14 16 and 8 _____

15 41 and 8 _____

16 20 and 13 _____

17 26 and 2 _____

18 50 and 20 _____

19 15 and 27 _____

20 63 and 35 _____

21 55 and 70 _____

Greatest common factor of three or four numbers

For the given three numbers, the method of finding GCF is same as the method of finding GCF of two numbers.

Example Find the greatest common factor of 18, 24 and 36.

The factors of 18 are $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, $\boxed{6}$, 9, 18.

The factors of 24 are $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, 4, $\boxed{6}$, 8, 12, 24.

The factors of 36 are $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, 4, $\boxed{6}$, 9, 12, 18, 36.

The common factors of 18, 24, and 36 are shown in \square .

The greatest among these factors is 6.

So GCF of 18, 24 and 36 is 6.

Example Compute greatest common factor of 68, 16, 76.

2	68
2	34
17	17
	1

2	16
2	8
2	4
2	2
	1

2	76
2	38
19	19
	1

Clearly $68 = 2^2 \times 17$, $16 = 2^4$, $76 = 2^2 \times 19$

Clearly $68 = 2^2 \times 17^1 \times 19^0$

$16 = 2^4 \times 17^0 \times 19^0$

$76 = 2^2 \times 17^0 \times 19^1$

Now, the GCF is the product of all above primes, each raised to the smallest of the two exponents.

G.C.F. of 68, 16, 76 = $2^2 \times 17^0 \times 19^0$

$$= 4 \times 1 \times 1 = 4$$

Work sheet 1.6

Compute the greatest common factor (GCF) of the following three number.

1 6, 78, 96 _____

2 45, 75, 30 _____

3 28, 68, 9 _____

4 98, 14, 28 _____

5 11, 66, 55 _____

6 13, 52, 39 _____

7 66, 16, 76 _____

8 $\text{gcd}(42, 63, 84)$ _____

9 $\text{gcd}(7, 28, 98)$ _____

10 $\text{gcd}(4, 37, 51)$ _____

11 $\text{gcd}(99, 11, 88)$ _____

12 The greatest common factor of 18, 30, 45 is
 (a) 3 (b) 5 (c) 6 (d) 9

13 The greatest common factor of 21, 36, 42 is
 (a) 2 (b) 3 (c) 6 (d) 7

Least common multiple

The least common multiple (LCM) or smallest common multiple of two integers is the smallest non-negative integer divisible by both the integers.

Example Find the LCM of 2 and 10.

Clearly multiples of 2 are 2, 4, 6, 8, $\boxed{10}$, 12.....

and multiples of 10 are $\boxed{10}$, 20, 30,

We can easily see that 10 is the smallest integer multiple of both.

So 10 is the LCM of 2 and 10.

Example Compute the LCM of 2 and 5.

Multiples of 2 are 2, 4, 6, 8, $\boxed{10}$, 12, 14.....

Multiples of 5 are 5, $\boxed{10}$, 15, 20,

Clearly 10 is smallest non negative integer divisible by 2 and 5 both.

So, 10 is the L.C.M of 2 and 5.

Example If $a = 270$ and $b = 252$ then compute LCM (a, b).

Clearly $a = 270 = 2 \cdot 3^3 \cdot 5$

$$b = 252 = 2^2 \cdot 3^2 \cdot 7$$

Now, match the prime divisors of the two integers by inserting missing prime raised to the power zero.

$$a = 270 = 2^1 \cdot 3^3 \cdot 5 \cdot 7^0$$

$$b = 252 = 2^2 \cdot 3^2 \cdot 5^0 \cdot 7^1$$

We know that GCD is the product of all above primes, each raised to the smallest of the two exponents.

For the LCM, we multiply primes raised to the largest of the two exponents.

Clearly $\text{LCM}(a, b) = \text{LCM}(270, 252)$

$$= 2^2 \cdot 3^3 \cdot 5^1 \cdot 7^1$$

$$= 3780$$

If a divides b , then $\text{GCD}(a, b) = a$
and $\text{LCM}(a, b) = b$

Example Compute GCD (21, 63) and LCM (21, 63).

Clearly 21 divides 63.

Example Compute LCM (0, 5).

LCM (0, 5) is undefined as by definition, LCM of two integers is the smallest non-negative integer that is multiple of both.

Further only multiple of 0 and 5 is 0. Thus, if the LCM of 0 and 5 had existed, it would have been 0. So if zero had been counted valid, then it would have been the LCM of all the numbers.

Work sheet 1.7

Find the least common multiple (LCM) of following pairs of numbers.

1 9, 27 _____

2 12, 27 _____

3 18, 48 _____

4 34, 68 _____

5 15, 75 _____

6 12, 32 _____

7 34, 51 _____

8 15, 65 _____

Fill in the following.

9 The multiples of 3 are _____, _____, _____, _____, _____

10 The multiples of 4 are _____, _____, _____, _____, _____

11 The common multiples of 3 and 4 are: _____, _____, _____, _____

12 The LCM of 3 and 4 is _____

13 The LCM of 16 and 40 is _____

14 The LCM of 35 and 75 is _____

15 The LCM of 13 and 65 is _____

16 The LCM of 10 and 30 is _____

17 The LCM of 42 and 70 is _____

18 The LCM of 50 and 75 is _____

19 The LCM of 15 and 90 is _____

20 The LCM of 30 and 81 is _____

Least common multiple of three numbers

Example Find the least common multiple (LCM) of 2, 3 and 12.

The multiples of 2 are 2, 4, 6, 8, 10, **12**, 14, 16, 18, 20, 22, **24**.....

The multiples of 3 are 3, 9, **12**, 15, 18, 21, **24**.....

The multiples of 12 are **12**, **24**, 36

The common multiples of 2, 3 and 12 are in bold.

The least common multiple of 2, 3 and 12 is 12.

Or LCM can be taken by prime factorization method:

2	2, 3, 12
2	1, 1, 6
3	1, 1, 3
	1, 1, 1

$$\text{LCM} = 2 \times 2 \times 3 = 4 \times 3 = 12$$

Work sheet 1.8

Find the least common multiple of the numbers.

1 14, 21, 7 _____

2 18, 29, 21 _____

3 14, 17, 12 _____

4 18, 30, 7 _____

5 11, 22, 17 _____

6 28, 12, 21 _____

7 25, 28, 7 _____

8 15, 30, 24 _____

9 13, 24, 20 _____

Find the LCM of the following by listing their multiples.

10 5, 10, 15 _____

11 28, 14, 21 _____

12 10, 20, 25 _____

Find the LCM by finding common prime factors.

13 60, 75, 120 _____

14 16, 24, 48 _____

15 18, 54, 72 _____

Find the LCM of the given numbers by prime factorisation method.

16 36, 27 and 18 _____

17 48, 64 and 120 _____

18 75, 150, and 275 _____

Find the LCM by division method.

19 70, 110, 150 _____

20 36, 60, 120 _____

21 21, 49, 63 _____

22 25, 45, 105 _____

23 36, 60, 120 _____

24 30, 150, 300 _____

Word problems on GCF and LCM

Example James has two ropes, one 20 feet long and the other 8 feet long. He wants to cut them equally with no wastage. What is the greatest length, that he can cut?

James has two ropes 20 feet and 8 feet in length. He wants to cut the rope so that he gets all equal lengths and no rope is wasted. Clearly for this situation greatest common divisor of 20 and 8 is to be found out.

$$20 = 5 \times \boxed{2 \times 2}, 8 = 2 \times \boxed{2 \times 2}$$

Clearly 4 is GCD. So 4 feet is the greatest length.

Example Nik baker's sells muffins in box of 6. Across town, Oven fresh sells muffins in packages of 4. If Linda wants to buy same number of muffins from each bakery for a party, what is the smallest number of muffins she will have to buy from each?

The least common multiple is the smallest whole numbers that is multiple of each of two or more numbers.

We will find LCM of 6 and 4.

$$\text{Prime factorization of } 6 = 2 \times 3 = 2^1 \times 3^1$$

$$4 = 2 \times 2 = 2^2 \times 3^0$$

For LCM we will multipl prime raised to the largest of the two exponents

$$\text{So } 2^2 \times 3^1 = 12$$

for LCM of 6 and 4 is 12. That means that the smallest number of muffins from each bakery is 12, because 2 boxes of 6 muffins from Nik bakers will have 12 muffins in total and 3 boxes of 4 muffins from Oven fresh will have 12 muffins in total.

Linda must buy atleast 12 muffins from each.

Example The traffic lights at three different road crossings change afer every 60 seconds, 90 seconds and 120 seconds respectively. If they all change simultaneously at 8:20:00 hrs when will they again change simultaneously?

To understand, set the two signals to change after every 3 seconds and 4 seconds respectively.

Clearly first signal changes after 3, 6, 9, 12 seconds and second signal changes after 4, 8, 12 seconds.

So, if the two signals change simultaneously, again they will change simultaneously after 12 seconds.

These 12 seconds were nothing but the LCM of 3 seconds and 4 seconds.

By the same method we can solve above problem. We have to find LCM of 58, 72 and 118.

$$\text{Clearly } 60 = 2^2 \times 3 \times \boxed{5}$$

$$90 = 2 \times \boxed{3^2} \times 5$$

$$120 = \boxed{2^3} \times 3 \times \boxed{5}$$

$$\text{LCM of } 60, 90, 120 = 2^3 \times 3^2 \times 5 = 360 = 6 \text{ min.}$$

So, after every 6 min, all the signals will change all together.

Hence, all signals will change at 8:26:00 hrs.

Work sheet 1.9

- 1 Paul has card board that mearsures 13 centimeters by 11 centimeters. If he wants to cut the smallest possible square, without wasting card board, how many centimeter long will each side of the square be?

- 2 One day, Mark and his friends had dinner while sitting at tables of 6. Another day they had lunch at tables of 11. What is the smallest number of people that could be in a group?

- 3 Sandra is a bird watcher. She notices an identical number of two types of birds in forest, Crows and Eagles. She always seem to observe crows in group of 7 and eagles in group of 10. What is the smallest number of crows that she could have seen?

- 4 At a birthday party with an equal number of boys and girls, boys are seated at tables of exactly 12 and girls are seated at tables of exactly 3. What is the minimum number of girls attending the party?

- 5 Find the least length of a rope which can be cut into whole number of pieces of length 45cm, 75 cm and 81 cm.

- 6 Find the greatest number of 4-digits which is exactly divisible by 40, 48 and 60.

- 7 What is the least number of saplings that can be arranged in rows of 12, 15 or 40 in each row?

- 8 210 oranges, 252 apples and 294 pears are equally packed in boxes so that no fruit is left. What is the biggest possible number of boxes needed?

- 9 Find the greatest number of 5-digits which on being divided by 9, 12, 24 and 45 leaves 3, 6, 18 and 39 as remainder respectively.

Classify numbers

Natural numbers: They are “counting numbers”.
Natural number begin at 1 and increment to infinity.

1, 2, 3, 4

Note: 1 is the smallest natural number.

Whole Numbers: Number 0 and numbers which are used while counting.

0, 1, 2, 3, 4

Integers: Integers are positive whole numbers, their opposite and zero.

..... -3, -2, -1, 0, 1, 2, 3

Rational Numbers: They can be written in the form

$\frac{p}{q}$ where $q \neq 0$. When written as decimals, rational numbers terminate or repeat.

$0, \frac{1}{2}, \frac{3}{4}, -1, 0.\bar{6}$ etc.

Irrational Numbers: They cannot be written as

$\frac{p}{q}$ ($q \neq 0$). When written as decimals, irrational numbers do not terminate or repeat.

$\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi, e, \sqrt{7} - 5, \frac{\sqrt{2}}{3}$ etc

Real Numbers: Real numbers include all rational and irrational numbers.

Note: 0 is whole number, an integer, a rational number, a real number but not a natural number and irrational number.

Integers are set of whole numbers and their negatives.

Whole numbers greater than 0 are called positive integers and the numbers which are less than 0 are called negative integers.

Like whole numbers, integers don't include fractions or decimals.

Note: Whole numbers contain natural numbers integers contains both natural numbers & whole number.

Note: Non negative integers are called whole numbers $\{0, 1, 2, 3, \dots\}$.

Note: Positive integers are called natural numbers $\{1, 2, 3, \dots\}$.

We denote the set of integers by z

So $z = \{ \dots -5, -4, -3, -2, -1, 0, 1, 2, 3, \dots \}$

Work sheet 1.10

- 1 Which of the following does not describe zero?
 - a) Real number
 - b) Rational number
 - c) Integer
 - d) Irrational number

- 2 Which of the following is rational number?
 - a) 5.888
 - b) $\sqrt{10}$
 - c) $\sqrt{7}$
 - d) π

- 3 Which of the following is natural number?
a) -7 b) 3 c) 6.444 d) π
- 4 Which of the following is irrational number?
a) $\sqrt{9}$ b) 3.111 c) $\frac{-7}{5}$ d) $\sqrt{32}$
- 5 Which of the following is an integer?
a) 11 b) π c) 62.536 d) $\frac{7}{4}$
- 6 Which of the following describe -100?
(a) whole number (b) integer (c) rational number (d) natural number
- 7 Which of the following are natural numbers?
(a) 65 (b) -90 (c) 0 (d) 15
- 8 Which of the following is a rational number?
(a) $\sqrt{2}$ (b) $\frac{-3}{5}$ (c) π (d) 4.2222....
- 9 Which of the following is an irrational number?
(a) 73 (b) $\frac{3}{8}$ (c) π (d) 0
- 10 Which of the following is not an integer?
(a) -70 (b) $\frac{5}{8}$ (c) 50 (d) 3
- 11 Which of the following are whole numbers?
(a) 100 (b) -50 (c) 88 (d) $\frac{5}{2}$

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State True / False.

12 π is neither an integer nor a whole number. _____

13 $\frac{42}{43}$ is an integer. _____

14 0 is a natural number. _____

15 0.04 is a whole number. _____

16 0, -9, 81, -273 are all integers. _____

17 $\frac{6}{2}$ is a nature number. _____

18 $\sqrt{3} + \pi$ is a rational number. _____

19 $1\frac{2}{4}$ is a rational number. _____

20 $\sqrt{\frac{1}{1}}$ is not a whole number. _____

Answers

Work sheet 1.1

1. 2, 3
2. (d)
3. (b)
4. (a), (b), (c), (d)
5. (b)
6. yes
7. (b)
8. (b)
9. (a)
10. 1, 2, 3, 4, 6, 12
11. 11, 22, 33, 44, 55
12. -15, -13, -45, -60
13. 1
14. No
15. 4, 9, 16, 25, 36, 49, 64, 81
16. number itself
17. 1
18. 3
19. 8, 16, 24
20. 24, 30, 36, 42, 48
21. 20, 40, 60, 80, 100, 120, 140, 160, 180
22. 3
23. 7
24. yes
25. yes
26. 1, 7, 13, 91
27. 152
28. -9, -18, -27, -36, -45
29. 112

Work sheet 1.2

1. yes
2. no
3. no
4. no
5. no
6. yes
7. no
8. yes

9. no
10. yes
11. yes
12. yes
13. no
14. no
15. yes
16. no
17. yes
18. no
19. yes
20. yes
21. no
22. no
23. no
24. yes
25. yes
26. no
27. yes
28. no
29. no
30. no
31. no
32. no
33. no
34. 1 or 3 or 5 or 7 or 9
35. 0 or 5
36. 0
37. 1 or 4 or 7
38. 1 or 4 or 7
39. 2
40. any digit from 0-9
41. 1 or 4 or 7
42. 2
43. 5
44. 0
45. 1
46. 1

Work sheet 1.3

1. 2, 3, 5, 7
2. 17, 19, 23

3. 53
4. 2, 3, 5, 7, 11
5. 31, 37, 41, 43, 47, 53, 59, 61, 67
6. $2 \times 2 \times 2$
7. 2×7
8. $2 \times 2 \times 3$
9. 3×7
10. 3×37
11. 2×43
12. Prime
13. Composite
14. Composite
15. Prime
16. Prime
17. Composite
18. Composite
19. Prime
20. Prime
21. Prime
22. Composite
23. Prime
24. Prime
25. Prime

Work sheet 1.4

1. 2×5
2. 2^4
3. $2^3 \times 3$
4. $2^3 \times 7$
5. 3^4
6. 7×13
7. 5^3
8. $2^2 \times 3 \times 11$
9. $2 \times 3 \times 11$
10. $3 \times 5 \times 7$
11. $2^2 \times 5^3$
12. 2×5^3

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13. $2^2 \times 3^2 \times 5$

14. $2^3 \times 5^2$

15. $2^3 \times 5^3$

16. $3^4 \times 5$

Work sheet 1.5

- 1
- 3
- 5
- 14
- 1
- 36
- 20
- 14
- 1
- 9
- 7
- 6
- 4
- 8
- 1
- 1
- 2
- 10
- 3
- 7
- 5

Work sheet 1.6

- 6
- 15
- 1
- 14
- 11
- 13
- 2
- 21
- 7
- 1
- 11
- (a)
- (b)

Work sheet 1.7

- 27
- 108
- 144
- 68
- 75
- 96
- 102
- 195
- 3, 6, 9, 12, 15
- 4, 8, 12, 16, 20
- 12, 24, 36, 48
- 12
- 80
- 525
- 65
- 30
- 210
- 150
- 90
- 810

Work sheet 1.8

- 42
- 3654
- 1428
- 630
- 374
- 84
- 700
- 120
- 1560
- 30
- 84
- 100
- 600
- 48
- 216
- 108
- 960
- 1650
- 11550
- 360

- 441
- 1575
- 360
- 300

Work sheet 1.9

- 1 cm
- 66 people
- 70 crows
- 12 girls
- 2025 cm
- 9840
- 120 saplings
- 18 boxes
- 99,714

Work sheet 1.10

- d
- a
- b
- d
- a
- b, c
- a, d
- b, d
- c
- b
- a, c
- True
- False
- False
- False
- True
- True
- False
- True
- False