23.1

Name:_____

Grade:_____

Ordered Pairs

The two objects placed in some particular order is called an ordered pair. If '***a***' and '***b***' are two objects, then the two different ordered pairs are **(*a*, *b*)** and **(*b*, *a*)**. We write the objects in particular order in parenthesis '( )', separating object by a comma (,) i.e. ***(x, y)***.

Note : We can write elements in place of objects, moreover in Mathematics living or non-living, both are considered as objects.

Example (2, 3), (Father, Son), (Son, Father) are examples of ordered pairs.
Clearly, (2,3) ≠ (3,2)
(Father, Son) ≠ (Son, Father)

Equality of ordered pairs

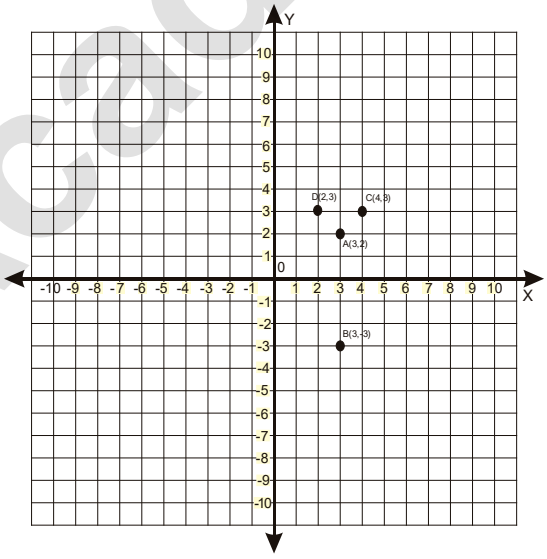
If (a, b) = (b, a) then a = b.
If (a, b) = (c, d) then a = c and b = d.

Plotting ordered pairs

All points on a grid (graph) can be expressed or identified, by using ordered pairs. We plot the first element of ordered pair on the horizontal line (the **x- axis**).

Find first element of the ordered pair **(*x, y*)** on *x*-axis and moving your finger up and down in a vertical line, still being at the same value on *x*-axis, to find the exact location of ordered pair, locate second number on the ***y*-axis** the vertical line moving your finger vertically. In the graph you can see point **A** is written as (3,2), **B** as (3, -3), **C** as (4, 3) and **D** as (2,3).

Note: First element of ordered pair is always plotted on ***x-axis*** and is called *abscissa* and second element is always plotted on ***y-axis*** and is called *ordinate*.



Work sheet 23.1

1. Plot the following ordered pairs on the graph

   A (1, 0)

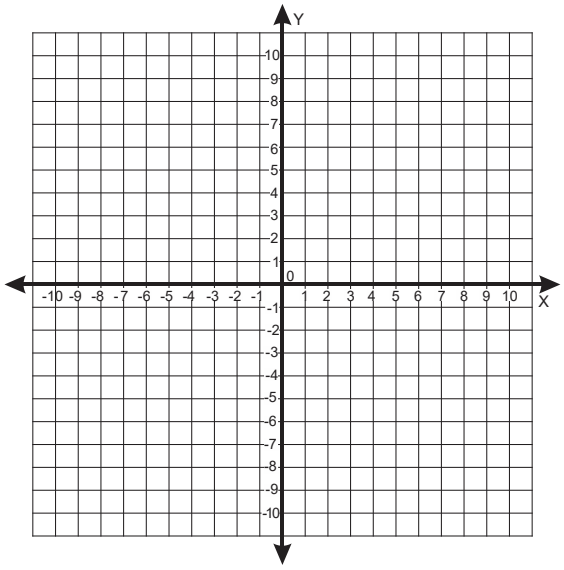
   B (0, -7)

   C (6, 6)

   D (5, 5)

   E (-5, 3)

   F (-4, -4)



Name:_____

Grade:_____

2 Plot the following ordered pairs on the graph

G (5, -1)

H (1/2, 1)

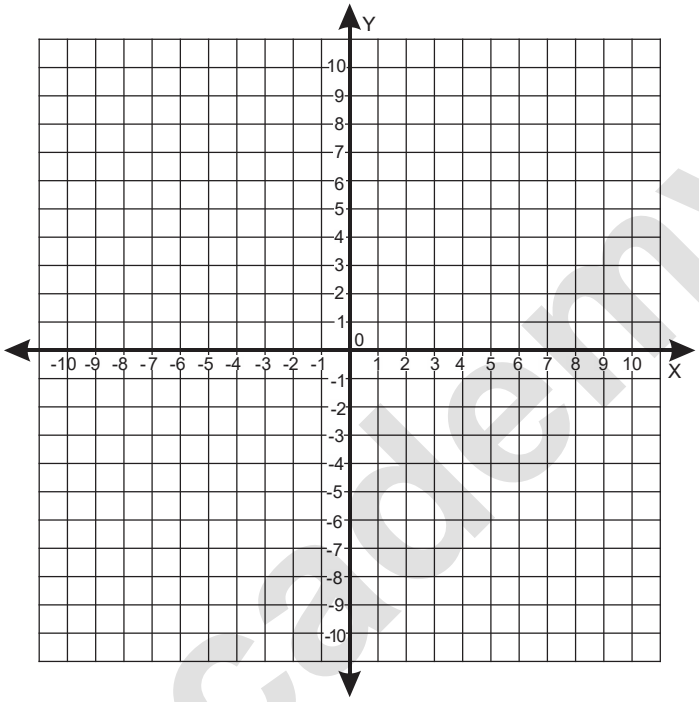
I (-3, 0)

J (0, -5)

K (-8, -8)

L (10, 1)

M (-1, 10)



3 Write ordered pairs for each marked point on the graph.

A _____

B _____

C _____

D _____

E _____

F _____

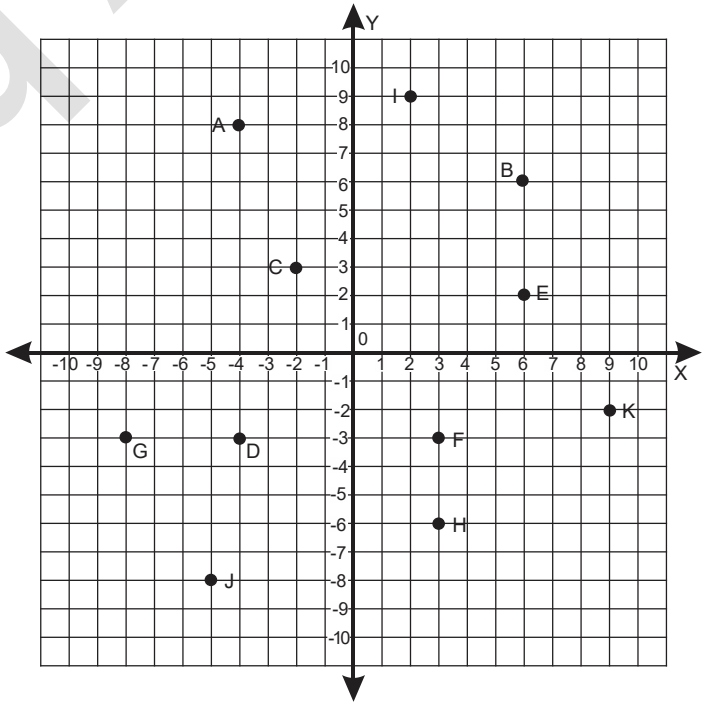
G _____

H _____

I _____

J _____

K _____



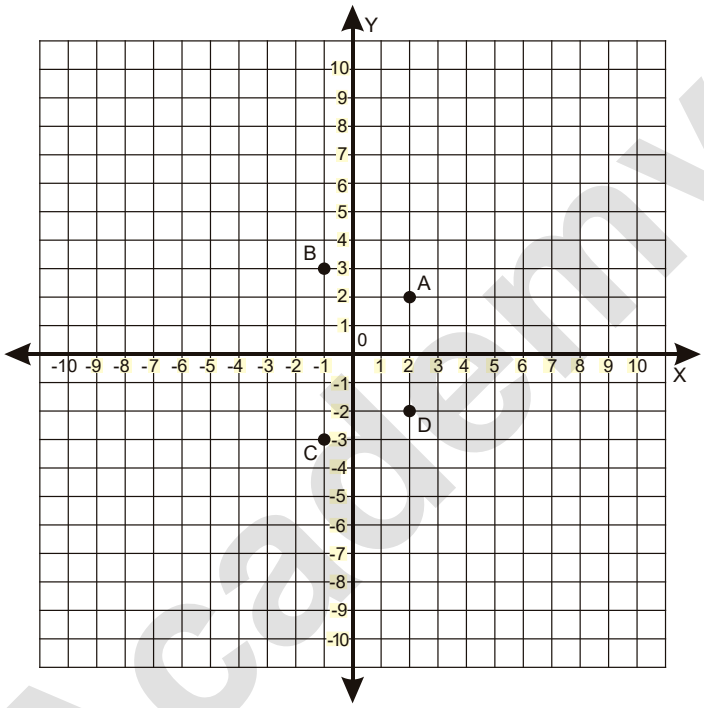
23.1

Name:_____

Grade:_____

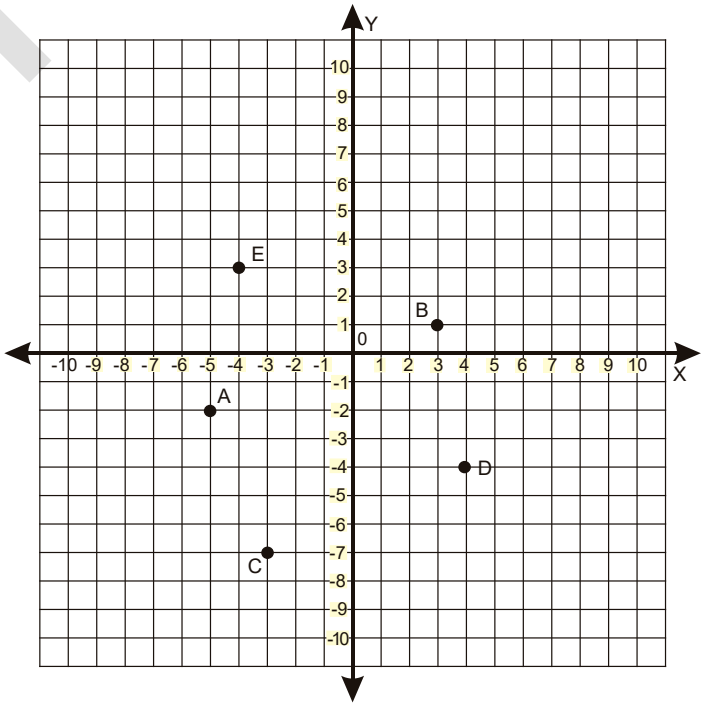
4 Which point has co-ordinates (-1, -3) ?

_____



5 Which point has co-ordinates (-5, -2) and (3, 1) ?

_____



Name:_____

Grade:_____

Relations and Functions: Introduction

Every point on the graph is represented by unique ordered pair or we can say that each ordered pair has unique representation on the graph. Ordered pair representing particular point is called co-ordinates of that point.

Grid (graph) is called coordinate plane and axes (x-axis, y-axis) are called coordinates axes.

Relations and Functions

How can we make a graph of an equation?

To draw a graph of an equation such as $y = 2x + 3$, we need number of ordered pairs (x,y) to plot on the grid (graph).

Easiest way to do this is to make a table.

To generate a table, substitute a number of values of x in the equation, and then solve it for y.

Example Consider equation $y = 2x + 3$. When choosing values of x to substitute in the equation, it is good to use whole number that will likely generate whole numbers for y. These are easier to plot.

Using $x = 0$ is always a good value. So $x = 0$ gives $y = 2(0) + 3 = 3$.

(0, 3) is an ordered pair. Simalarly we can take other values of x to get corresponding values of y. We can show this in table.

Input	x	0	1	-1	2	-2	3
Out put	y	3	5	1	7	-1	9
Ordered Pear	(x,y)	(0, 3)	(1, 5)	(-1, 1)	(2, 7)	(-2, -1)	(3, 9)

From above table we can see that

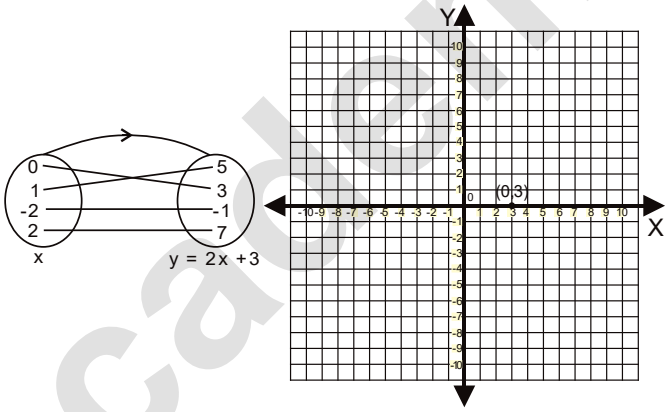
$x = 0$ is related to $y = 3$ by equation $y = 2x + 3$.

A relation between x and y is a collection of ordered pairs .

A **relation** is collection of inputs and output written in ordered pairs as (input, output) **.**

From above example we can clearly see x is related to y by relation $y = 2x + 3$ and from the table we get collection of ordered pair (x,y) or $(x,2x + 3)$

Relation can also be represented by tables, graphs and mapping diagrams.



We can denote relation by 'R', clearly R is defined by ordered pairs (0,3) (1,5) (-2,-1) (2,-7),-----

A function is a relation with one (unique) output for each input.

A **function** is a special type of relation that pairs each domain value with exactly one range value.

The **domain** of a relation is the collection of first coordinates (or x-values) of the ordered pairs whereas the **range** of a relation is the set of second coordinates (or y-value) of the ordered pairs. In above example 0, 1, -2, 2 is domain and range is 5,3,-1,7.

The Vertical - line test

The vertical - line test can be used to visually determine whether a graphed relation is a function .

Consider a table of ordered pairs and plot them on the graph.

23.2

Name:_____

Grade:_____

Table A

x	-2	-1	0	1	2	3
y	-4	-3	-1	1	3	5

Graph A

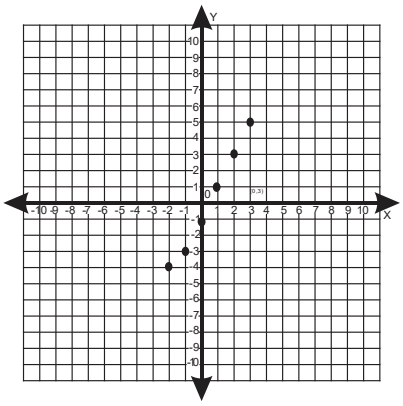
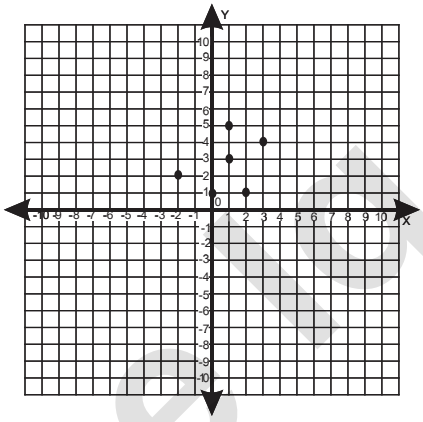


Table B

x	-2	1	0	1	2	3
y	2	3	1	5	1	4

Graph B



Look at the value in table A and table B. Is every x-value paired exactly one y - value?

We can see in table B that some of the x - values are paired with more than one y-value.

Now draw a vertical line from each point of the graph and see whether it passes through more than one point or not.

In graph B we can see that vertical line passes through (1,3) as well as (1,5)

If vertical line passes through more than one point then, that relation is not a function.

Clearly from above graphs A and B, graph A is a graph of a function and graph B is a relation but not a function or not.

You can use the ***vertical line test*** on a graph to check whether a relation is a function.

If it is possible to draw a vertical line that intersects the graph more than once, then there is an x-value, that is paired with more then one y-value. So, the *relation is not a function.*

If it is impossible to draw a vertical line, that intersects the graph more than once, then each of the x-value that is paired with only one y-value. So, the *relation is a function*.

Work sheet 23. 2

1 Look at the table :

x	0	6	5	4
y	2	17	20	5

Is this relation a function ?

Yes ☐ No ☐

2 Which of the relations is a function?

(a) (19, 2) (20, 2) (19, 4) (13, -2) ☐

(b) (7, -9) (20, -6) (20, 9) ☐

(c) (13, 3) (-2,17) (6, 16) (8, -16) ☐

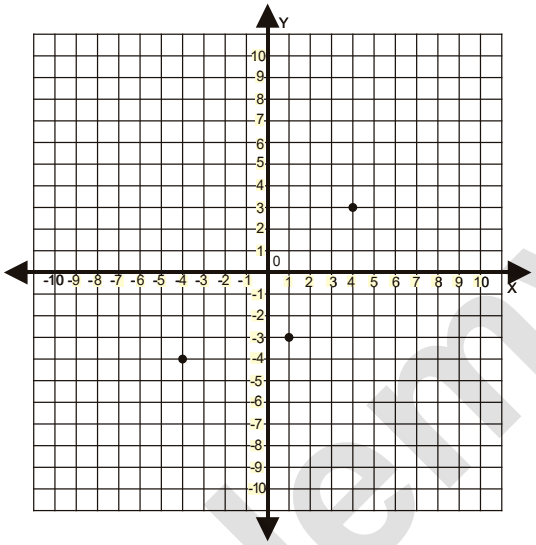
(d) (17, -5) (5, 20) (17, -12) ☐

Name:_____

Grade:_____

23.2

Look at the graph



3 Is this relation a function ?

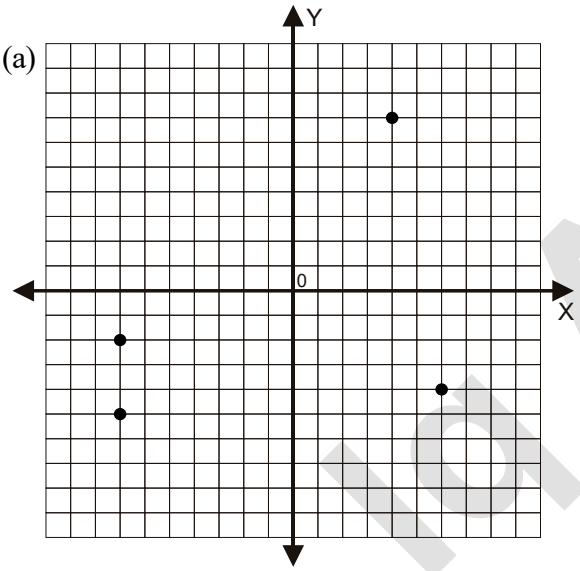
Yes ☐ No ☐

4 Observe the ordered pairs :
(-17, 6)
(7, 1)
(0, 15)
Is this relation a function ?

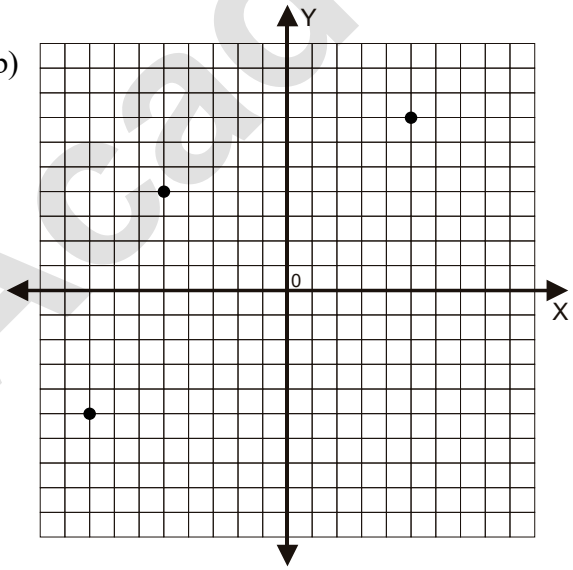
Yes ☐ No ☐

5 Which of these relations is a function ?

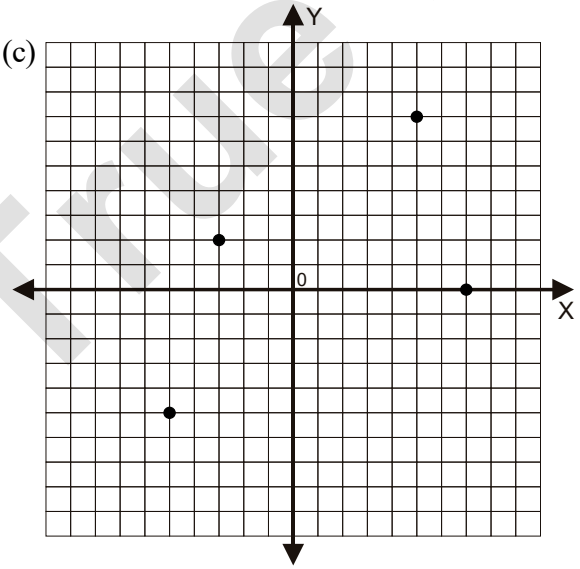
(a)



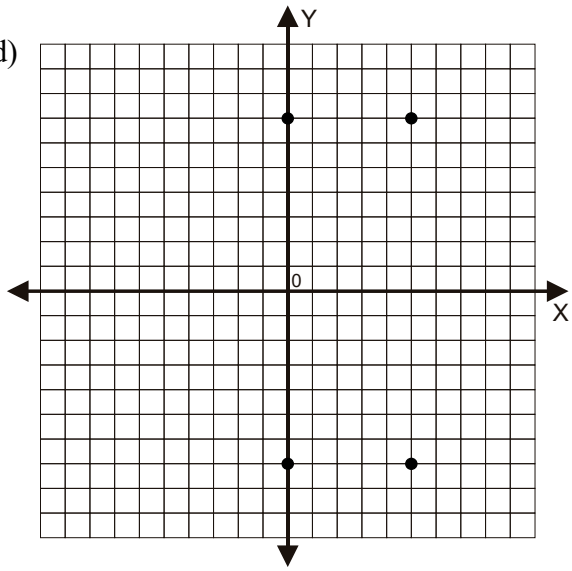
(b)



(c)



(d)



23.2

Name:_____

Grade:_____

6 Which of these relations is a function?

(a) (17, -18), (7, 18), (7,5)

(b) (1, 4), (8, 2), (8, -4), (1, 11)

(c) (14, -3), (11, -10), (14, 10)

(d) (14, 18), (7, -10), (15, 10)

_____

7 Which of these relations is a function ?

(a)

x	12	11	0	12	12
y	18	-18	5	25	-25

(b)

x	2	5	-1	0	6
y	3	3	1	1	1

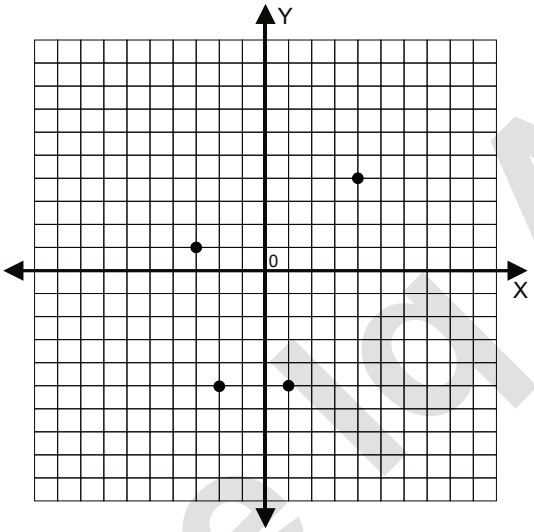
(c)

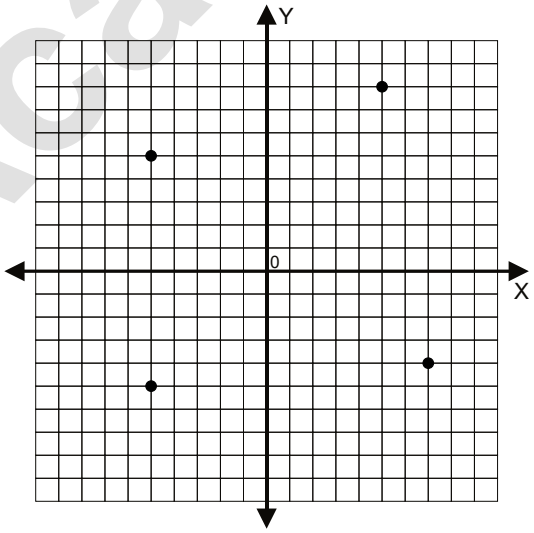
x	1	2	2	3	5
y	2	3	-1	5	-5

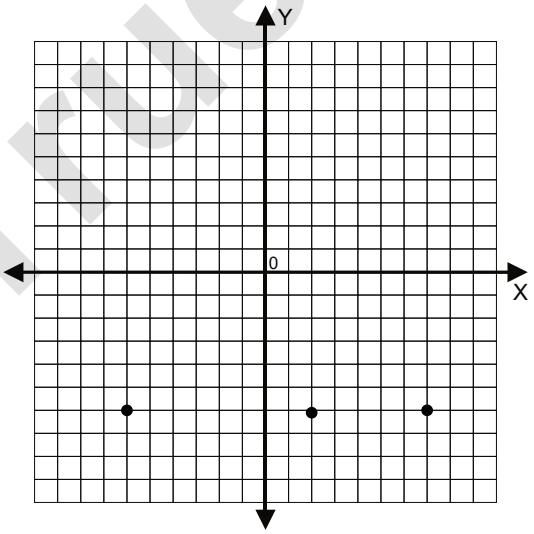
(d)

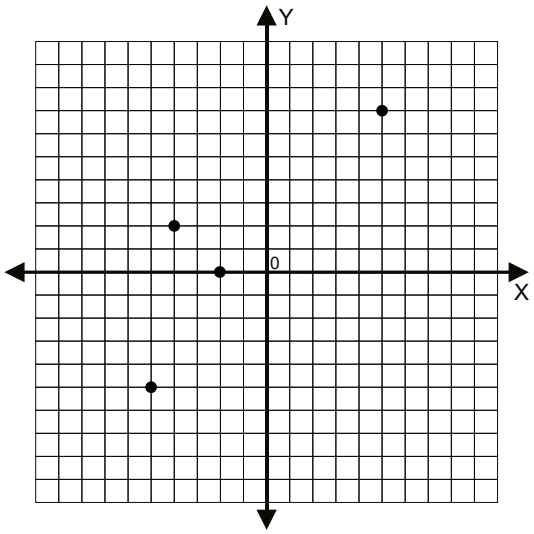
x	0	2	4	-3	0
y	1	3	5	-1	-1

8 Which of these relations is not a function ?







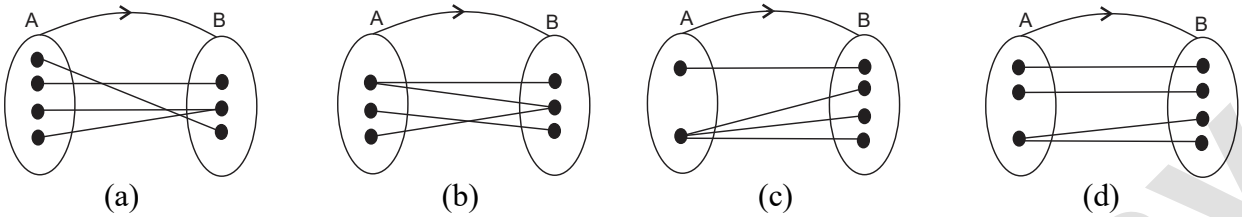


Name:_____

Grade:_____

23.2

9 Which of the following relation mapping between A and B is a function?



(a) (b) (c) (d)

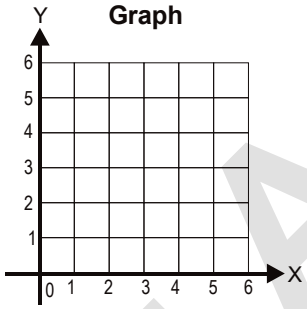
_____

10 Express the relation for the ordered pairs, (1,5), (7,3), (3,1), (4,2), (5,4) as a table, as a graph, and as a map ping diagram.

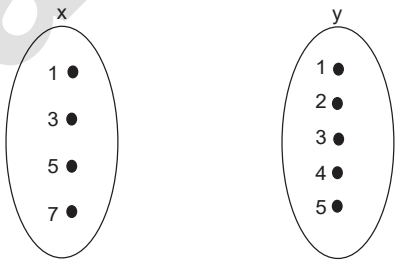
Table

x	y
1	
3	
5	
7	

Graph



Mapping diagram



Is above relation a function? Yes ☐ No ☐

11 Give the domain and range of each relation.

School trip	
x (Student)	y (buses)
70	2
68	2
125	3

Domain : _____

Range : _____

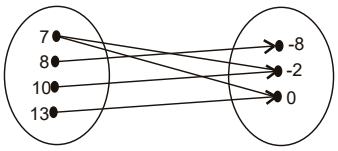
This relation is not a function. True ☐ False ☐

23.2

Name:_____

Grade:_____

12



Domain : _____

Range : _____

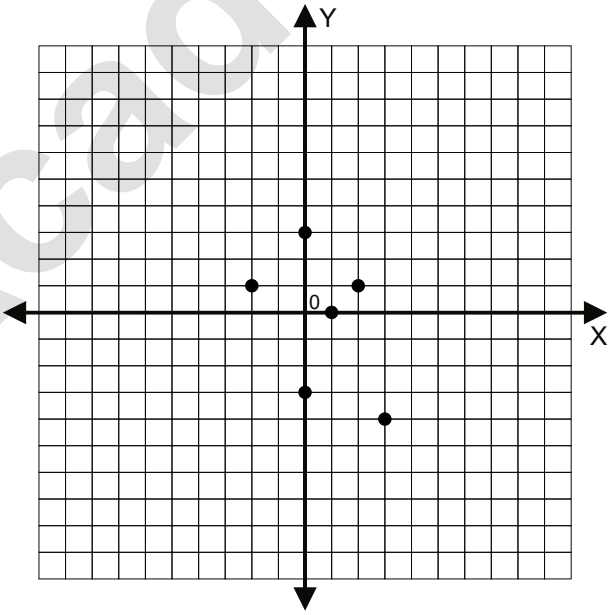
Is this relation a function ?

Yes ☐ No ☐

13 Write ordered pair for a relation represented in the given graph

(____, ____), (____, ____), (____, ____)

(____, ____)(____, ____), (____, ____)



Domain : _____

Range : _____

Is this relation a function ?

Yes ☐ No ☐

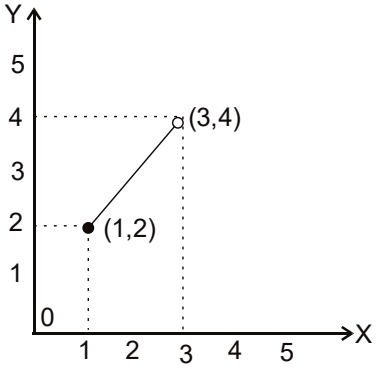
14 Find the domain and range of a relation shown by following graphs.

Domain : _____

Range : _____

Is this relation a function ?

Yes ☐ No ☐

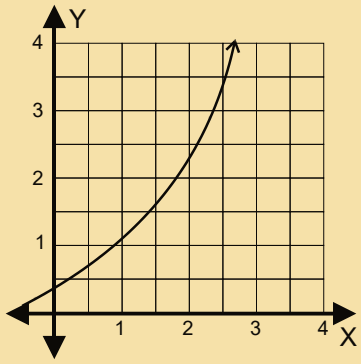


Name:_____

Grade:_____

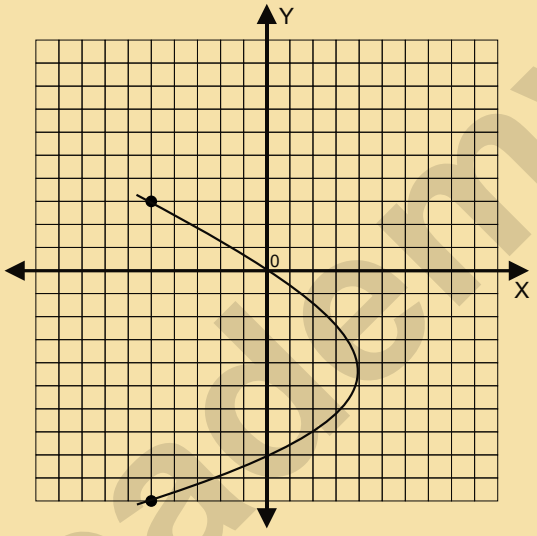
Identify function from the graphs

Example Look at the graph :



It is not possible to draw a vertical line that intersect the graph more then once so the relation is a function

Look at the graph

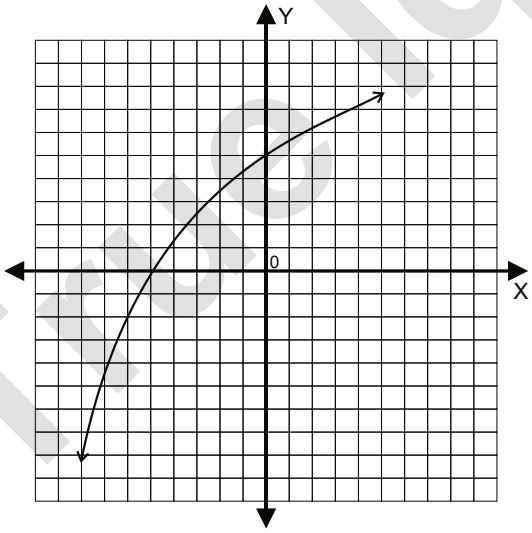


We can see that red line intersects the graph in two places, so the relation is not a function.

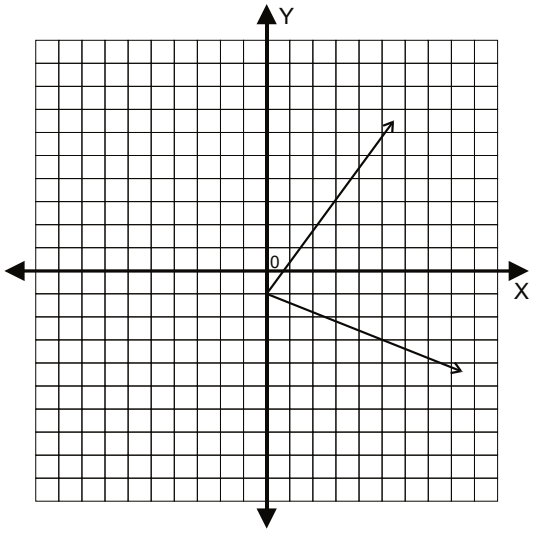
Work sheet 23.3

By looking at the graphs, tell whether the relation is function or not?

1



2



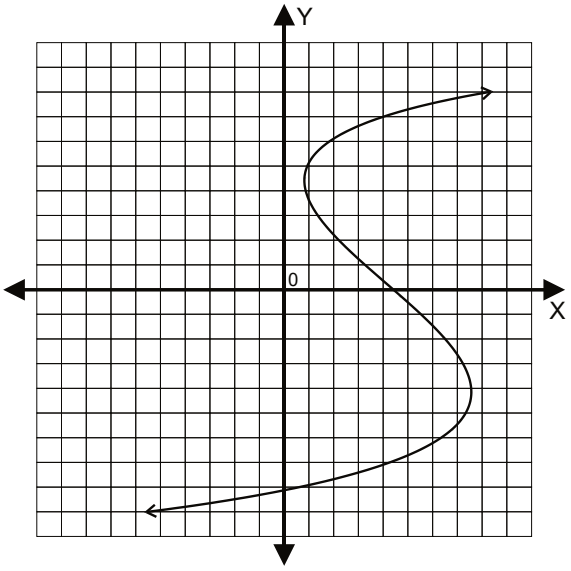
_____ _____

23.3

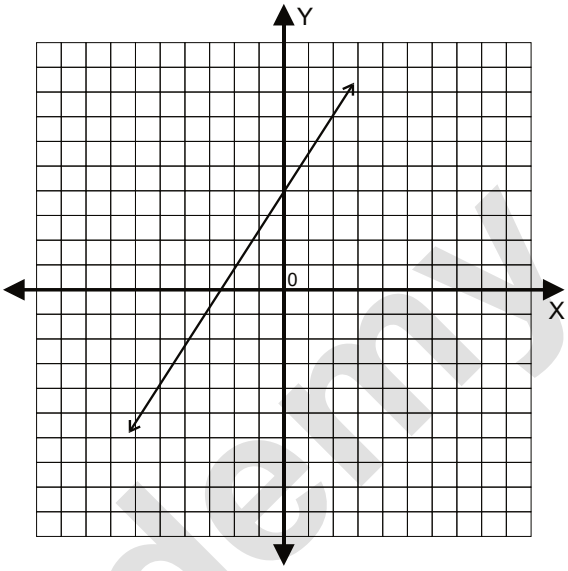
Name:_____

Grade:_____

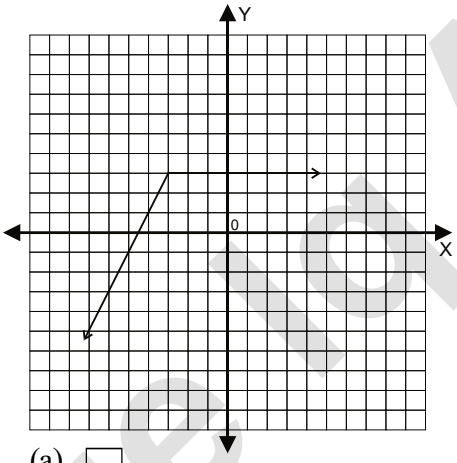
3



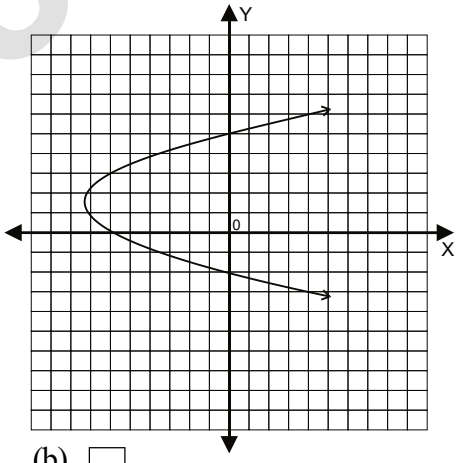
4



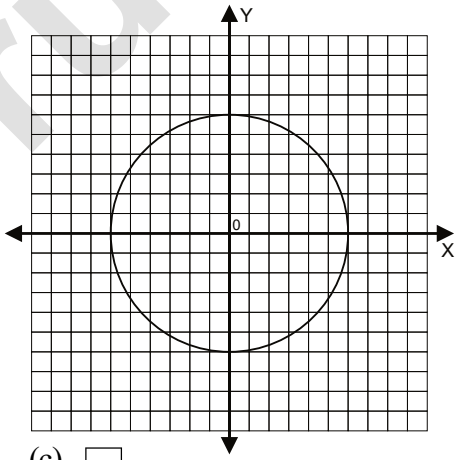
5 Which of these relations is a function ?



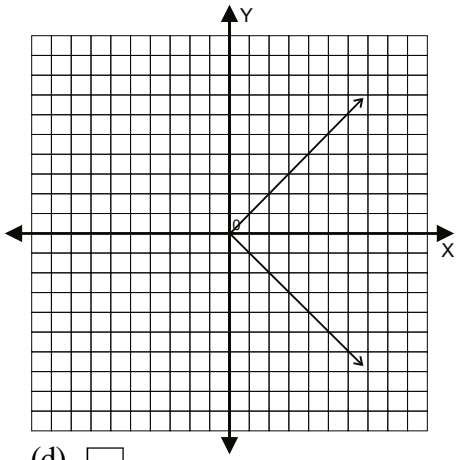
(a) ☐



(b) ☐



(c) ☐



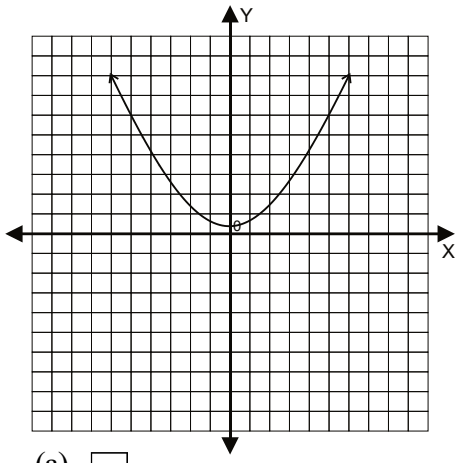
(d) ☐

Name:_____

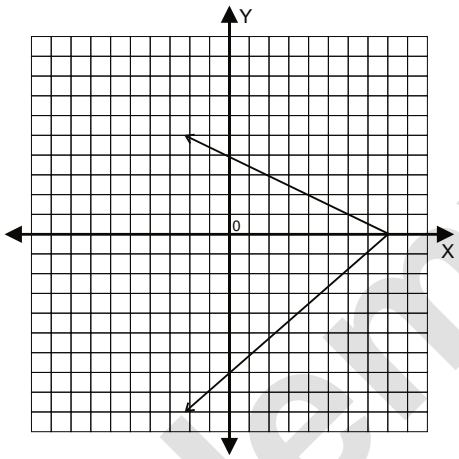
Grade:_____

23.3

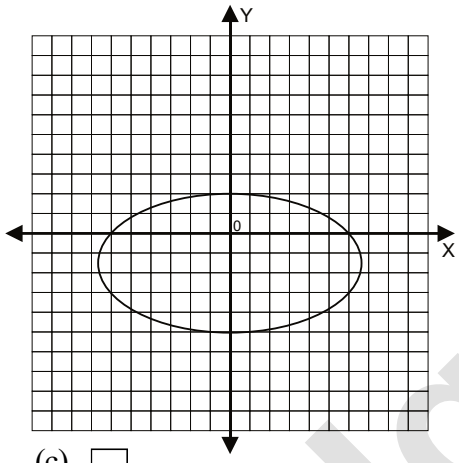
6 Which of these relations is a function ?



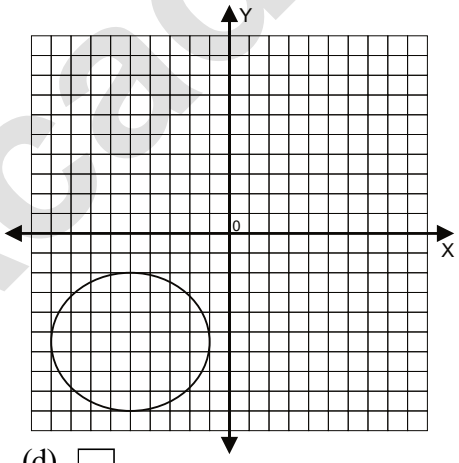
(a) ☐



(b) ☐

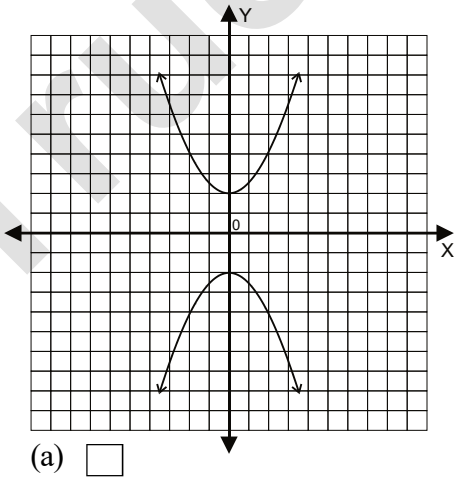


(c) ☐

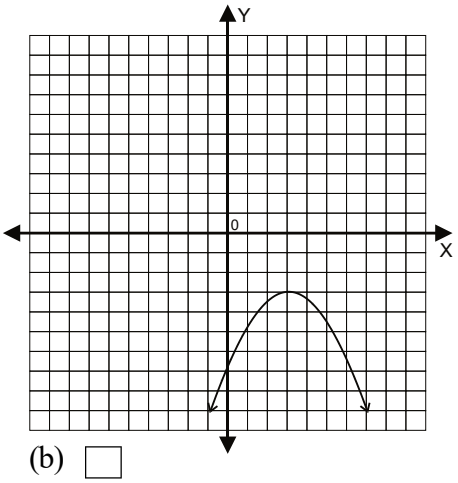


(d) ☐

7 Which of these relations is a function?



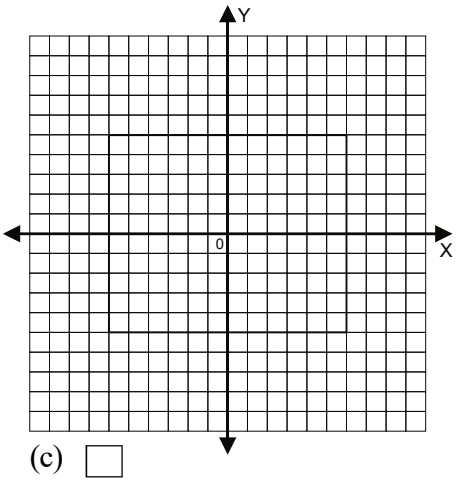
(a) ☐



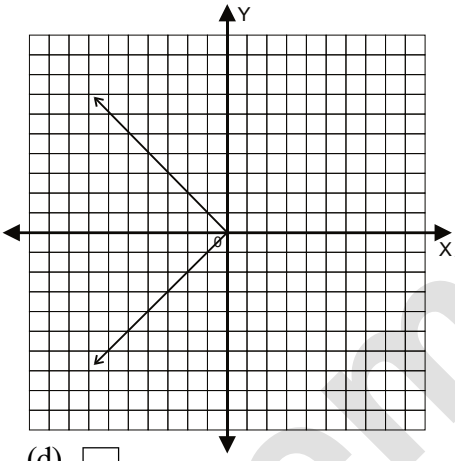
(b) ☐

Name:_____

Grade:_____

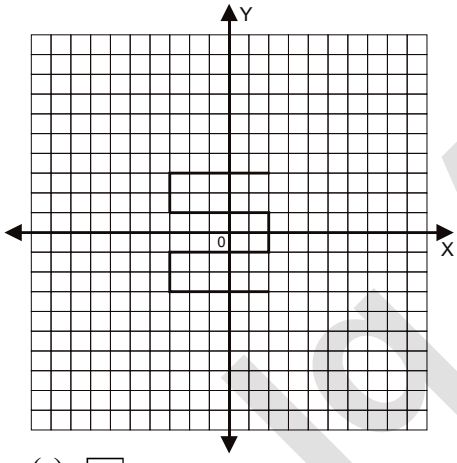


(c) ☐

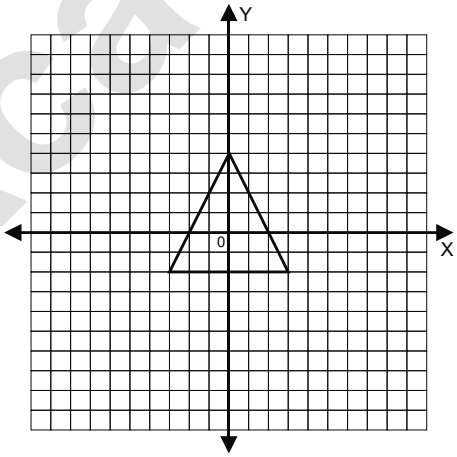


(d) ☐

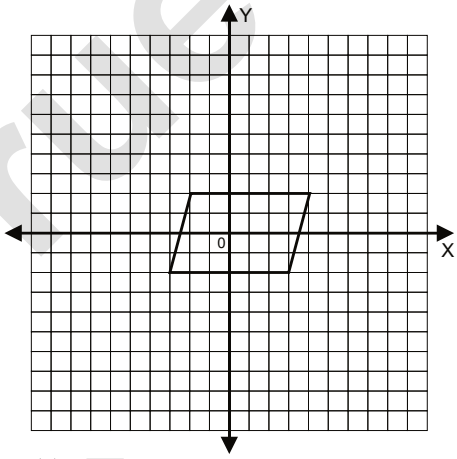
8 Which of these relations is a function?



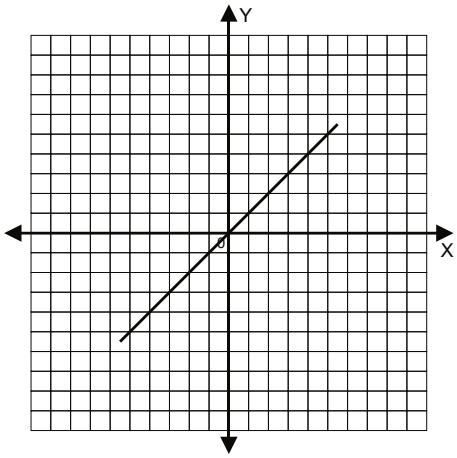
(a) ☐



(b) ☐



(a) ☐



(b) ☐

Name:_____

Grade:_____

23.4

Is (x, y) a solution for linear equation?

Is an ordered pair (x,y) a solution of linear equation or linear function ?

If a function is linear then change in y over the change in x is always constant.

Example Consider a function that contains the following points or ordered pairs.

x	y
1	8
2	13
3	18
4	23
5	28

Is this function linear or non linear? To be a linear function, for any given change in x we must have change in y and that change must be constant (same value) .

We can see in above ordered pairs, for fix change in x there is constant change in y . We can see that x is changing by 1, the change in y always be the same. If change is not constant, it is not a *linear function* or it is *a non-linear function.*

Clearly above points represents a linear function .
We can also say that a function whose graph is a straight line is a linear function. Linear function has the following form,
$y = a + bx$, where a and b are constants, $b \neq 0$ and x, y are variables.
A point (*ordered pair*) satisfies a linear equation, if plugging x and y results in *true* statement. We can say that (x,y) is solution of linear equation $y = (a + bx)$.

Example: Does the point (3,4) satisfy the equation $y = 3x - 5$?
We know that in the ordered pair (3,4), 3 is the x-value and 4 is the y-value.
Plug $x = 3$ and $y = 4$ into the equation.
$y = 3x - 5$
$4 = 3 \times 3 - 5$
$4 = 9 - 5$
$4 = 4$, yes
So the point (3, 4) satisfies the equation $y = 3x - 5$.

Work sheet 23.4

1. Does the point (6,0) satisfy the equation $y = x + 5$? Yes ☐ No ☐

2. Is (8, 2) point lies on the equation $y = x$? Yes ☐ No ☐

3. Is (7, 1) a solution to the equation $y = 3x + 1$? Yes ☐ No ☐

4. Is (4, 4) solution to the equation $y = -x$? Yes ☐ No ☐

23.4

Name:_____

Grade:_____

5. Does (1, 0) make the equation $y = 2x - 1$ true ? Yes ☐ No ☐

6. Does the point (0, 0) make the equation $y = 8x$ true ? Yes ☐ No ☐

7. Does (2, 0) make the equation $y = x + 3$ true? Yes ☐ No ☐

8. Does the point (8, 2) satisfy the equation $y = 3x$? Yes ☐ No ☐

9. Is (2, 1) solution the equation $2x - 3y = 1$? Yes ☐ No ☐

10. Consider the following points on a graph of a function.
(1, 3), (2, 5), (3, 8), (4, 9), (5, 11)
Is this function a linear or Non-linear ? _____

11. Is (1, 2) solution of equation $x + y = 1$? Yes ☐ No ☐

12. Is (7, 9) solution of equation $x + y = 2$? Yes ☐ No ☐

13. Is (1, 8) solution of equation $y = 2x + 3$? Yes ☐ No ☐

14. Is (1, 3) solution of equation $y = 3 - 2x$? Yes ☐ No ☐

15. Is (4, 13) solution of equation $y = 7 + 2x$? Yes ☐ No ☐

16. Is (7, 20) solution of equation $y = 3x - 1$? Yes ☐ No ☐

Name:_____

Grade:_____

Independent and dependent variables

We know that a **variable** is a quantity that may change within the context of a mathematical problem or experiment. The letters x, y and z are common symbols used for variables.

The variable that can be changed or controlled in a given equation or experiment is called **independent variable**. It is the variable that is changed or controlled in a scientific experiment to test the effects on the dependent variable.

The value of result that is due to the independent variable is called **dependent variable**. It is the variable being tested and measured in a scientific experiment.

Example If a scientist conducts an experiment to test the theory that a vitamin could extend a person's life-expectancy then, the *independent variable* is the amount of vitamin that is given to the person . This is controlled by the experimenting scientist. The *dependent variable* is the outcome of the experiment and being affected by the independent variable is life span.

Example A scientist wants to see if the intensity of light has any effect on a moth being attracted to the light. The intensity of the light is controlled by the scientist. This would be the *independent variable.*

How the moth reacts to the different light levels (distance to light source) would be the *dependent variable*.

Example Consider equation

$$y = 2x - 1$$

For $x = 1$, we get $y = 2 \times 1 - 1 = 1$

For $x = 2$, we get $y = 2 \times 2 - 1 = 3$

Clearly value of y depends on value of x. Here we can say that x is an independent variable and y is a dependent variable.

The difference is that the value of the independent variable is controlled by the experimenter, while the value of the dependent variable only changes in response to change in the independent variable.

Independent variable and Dependent variable

The input of a function is the independent variable. The output of a function is the dependent variable. The value of the dependent variable depends on the value of the independent variable or is a function of, the value of the independent variable.

Example: The faster Sam runs, the quicker he gets to school.

The time it takes Sam to get school depends on the speed he runs.

Dependent variable : Time.

Independent variable : Speed

Example: Suppose Jack is a dog walker and charges $10 per hour.

Clearly here x is independent variable and y is dependent variable.

Write an equation using two different variable to show this relationship.

$$\underbrace{Amount\ earned}_{y} \underbrace{=}_{=} \underbrace{\$10}_{10} \underbrace{\ times\ }_{\bullet} \underbrace{the\ number\ of\ hours\ worked}_{x}$$

Jack can use the equation $y = 10 \bullet x$ to find how much money he will earn for any number of hours he works.

If x is the independent variable and y is the depen dent variable, then function rotation for y is f(x), read "f of x" where f is the function.

23.5

Name:_____

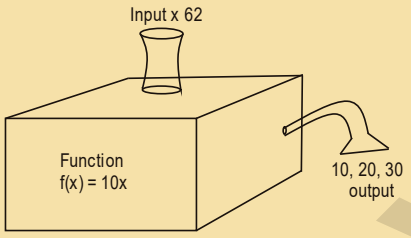
Grade:_____

The dependent variable	is	a function of	the independent variable

y	is	a function of	x

y	=	f	x

Since y = f (x), Jack earning, y = 10x, can be rewrittin in function naotation by substituting find for y : f(x) = 10x. You can think of a function as an input-output machine. For Jack's earnings f(x)=10x, if you input a value x, the output is 10x .



Example: For f(x) = 3x -1, Find f(x) .

When x =1, x = 0, and x = 2

For x = 1, f(1) = 3 × 1-1 = 2

For x = 0, f(0) = 3 × 0 -1 = -1

For x = 2, f(2) = 3 × 2 - 1 = 5

Work sheet 23.5

1. Identify the independent and dependent variables in each solution

   An employee recieves 2 vacation days for every month worked .

   Dependent variable : _____

   Independent variable : _____

2. For every day of study chance for scoring good marks increase.

   Dependent variable : _____

   Independent variable : _____

3. For every mile travelled in a rented car, there is an increase of fare by $3.

   Dependent variable : _____

   Independent variable : _____

4 The more Martha watched television, there are more tired eyes of Martha.

Dependent variable : _____

Independent variable : _____

5 For each mile walked, Mark requires one glass of water more to quench his thirst.

Dependent variable : _____

Independent variable : _____

6 For each hour of rehersal for the dance performance, Lucy scores 2 points more in dance competition.

Dependent variable : _____

Independent variable : _____

7 For every hour of overtime worked, Smith gets an extra bonus of $10.

Dependent variable : _____

Independent variable : _____

8 The more are the sun hours in a day, the more hours the electricity supply from the solar battery.

Dependent variable : _____

Independent variable : _____

9 The more Justin plays in a day, the more his legs gets weak.

Dependent variable : _____

Independent variable : _____

23.6

Name:_____

Grade:_____

Rate of change

A rate of change is a rate that describes how a quantity changes in a relation to change in time.

$$\text{Rate of Change} = \frac{\textit{change in value of quantity}}{\textit{Change in time}}$$

Example: Find the rate of change from the given data.

Driving Time (h) x	Distance travelled(mi) y
2	80
4	160
6	240

$$\text{Rate of Change} = : \frac{\textit{Change in } y}{\textit{Change in } x}$$

$$= \frac{\text{Change in distance}}{\text{Change in time}}$$

$$= \frac{160-80}{4-2} = \frac{80}{2} = 40$$

This means a vehicle is travelling at a rate of 40 miles per hour.

Further, rate of change is the rate that describes how one quantity changes in relation to another quantity.

If x is independent variable *and* y is dependent variable then rate of change $= \frac{\textit{change in } y}{\textit{change in } x}$.

Work sheet 23.6

1. Table shows the cost of mailing 1- ounce letters in different years. Find the rate of change in cost for time interval 2004-2008.

Year	1990	1992	1996	2002	2004	2006	2008
Cost ($)	25	29	27	26	29	37	28

_____

2. Tables shows the bank balance and number of days of a month. Find rate of change of bank balance from10 days to 15 days.

Days	1	5	10	15	20	30
Bank Balance ($)	550	280	270	190	160	120

_____

3. Given table of distance left from Clara's house, when Audrey left for her house and the corresponding speed of Andrey's Car.

Distance left (miles)	25	20	10	5	2
Speed of Andrey's Car (mph)	31	34	37	24	18

Find the rate of change of speed when distance changed from remaining 17 miles to when remaining distance was 7 miles ?

_____

Name:_____

Grade:_____

23.6

4 Find the rate of change of price from the year 2016 to 2020.

Year	Price $
2012	17
2014	18
2016	20
2018	23
2020	24

_____

5 From the given table A what is rate of change of distance travelled from 4 hrs to 6 hrs?

Time (hrs)	Distance (mi)
1	0.5
3	1.5
4	2.5
6	4.5
10	6

Table A

_____

6 From the given table A what is rate of change of distance travelled at 3 hours and at 6 hours ?

_____

7 From the table B what is rate of change of price of commodity A from 2000 to 2005?

Year	Price of A	Price of B
1990	15	20
1995	15	21
2000	17	21
2005	23	23
2010	25	23
2015	26	27

Table B

_____

8 From the table B what is rate of change of price of commodity B from 2005 to 2010?

_____

9 Observe table B and find from the year 1995 to 2005 which commodity is having greater rate of change of price?

A ☐ B ☐

10 Observe the table B, find in which years the rate of change of price is greater for commodity B.

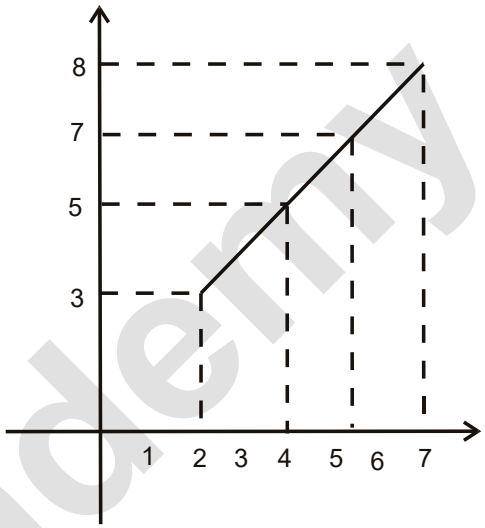
(a) 1995-2000 ☐ (b) 2005 - 2010 ☐ (c) 2010 - 2015 ☐

23.6

Name:_____

Grade:_____

Observe the following graph and answer the following.



11 What is rate of changes of y, when x change from 4 to 7 ?

_____

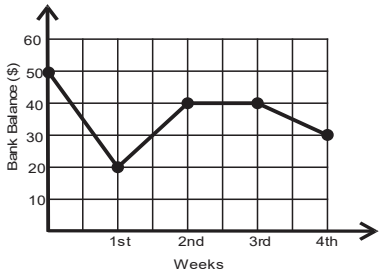
12 What is rate of changes of y, when x change from 5 to 7 ?

_____

13 What is rate of changes of y, when x change from 2 to 5 ?

_____

Following graph shows the bank balance of Martin for October at the end of all 4 weeks.



14 What is rate of change of amount in bank from 2nd to 4th week of October?

_____

15 During what time interval Martin deposits some amount in his bank account ?

(a) 1st Week ☐ (b) 2nd Week ☐ (c) 3rd Week ☐

16 During which week Martin withdrew maximum amount from his bank ?

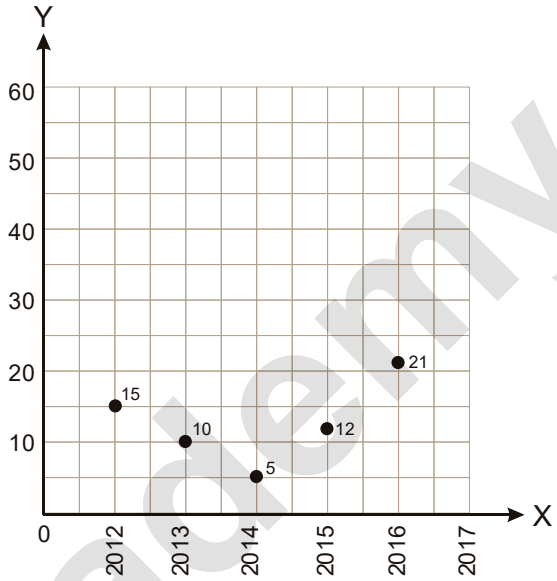
(a) 1st Week ☐ (b) 2nd Week ☐ (c) 3rd Week ☐

Name:_____

Grade:_____

23.6

The graph shows number of prizes won by a school annually



17 What is rate of change of number of prizes from year 2013 to 2015 ?

_____

18 What is rate of change of number of prizes from year 2012 to 2016 ?

_____

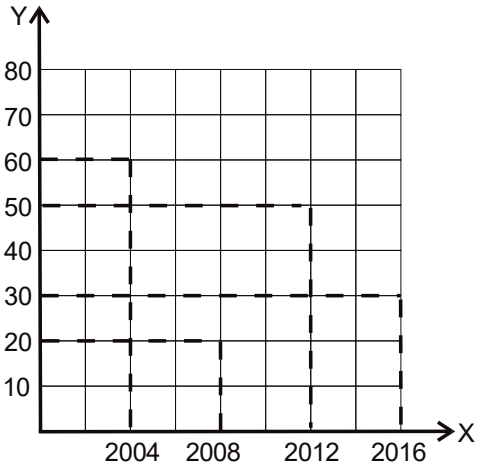
19 In which year the rate of change of number of prizes was maximum ?

(a) 2013 - 2014 ☐ (b) 2014 - 2015 ☐ (c) 2015 - 2016 ☐

20 In which year the rate of change is the least ?

(a) 2014 - 2015 ☐ (b) 2013 - 2014 ☐ (c) 2015 - 2016 ☐

The graph shows no. of medals won by USA in olympics.



21 What is rate of change of number for medals from year 2008 to 2016.

_____

22 What is rate of change of number of medals from year 2004 to 2016.

_____

23.7

Name:_____

Grade:_____

Constant rate of change & slope

The constant rate of change is a ratio that describes how one quantity changes in relation to another quantity. If x is the independent variable and y is the dependent variable, then

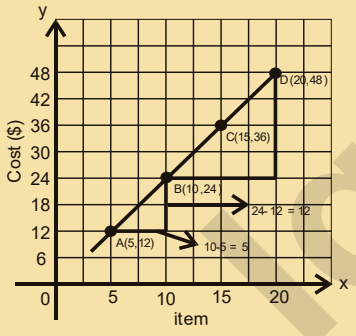
$$\text{Rate of change} = \frac{change\ \ in\ \ y}{change\ \ in\ \ x}$$

or we can say if one quantity changes in relation to the other by a constant factor then it is constant rate of change.

Example: Find the constant rate of change between the quantities in the table as shown.

Item	5	10	15	20
Cost ($)	12	24	36	48

The cost increases by $12 for every 5 items.



We can find slope of line by using any two points on a line. Consider point A and B (we may consider any two points from A , B, C or D).

Slope of line through A and B =

$$\frac{rise}{run} = \frac{Change\ in\ y}{Change\ x} \text{ or}$$

$$\frac{Vertical\ Change}{horizonta\ Change} = \frac{24-12}{10-5} = \frac{12}{5} = \frac{\$2.40}{1\ item}$$

So the constant rate of change = $ 2.40 per item.

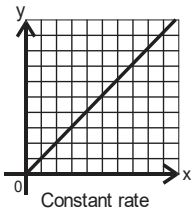
If we consider any two points on graph say (x_1, y_1) and (x_2, y_2).

Then constant rate of change =

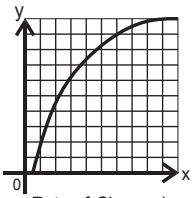
slope of line joining A and B =

$$\frac{rise}{run} = \frac{Change\ in\ y}{Change\ in\ x} \quad \frac{(Vertical\ Change)}{(horizontalChange)}$$

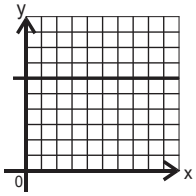
$$= \frac{y_2 - y_1}{x_2 - x_1}$$



Constant rate of change



Rate of Change is not constant



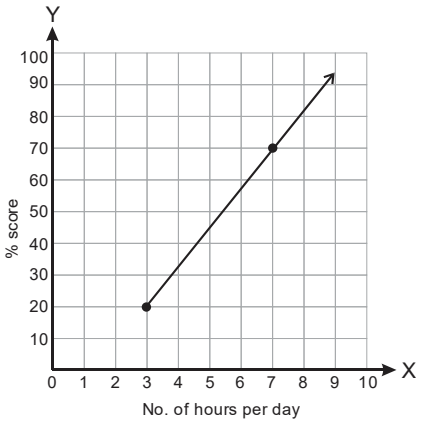
No rate of Change

Work sheet 23.7

Graph shows the number of hours Roxanne study per day and % of marks she scored in exams she attempted ?

1 Find the rate of % score per hour for Roxanne.

_____

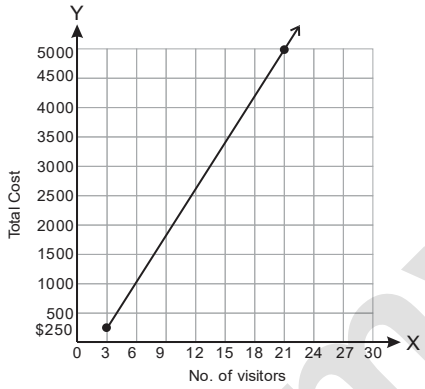


Name:_____

Grade:_____

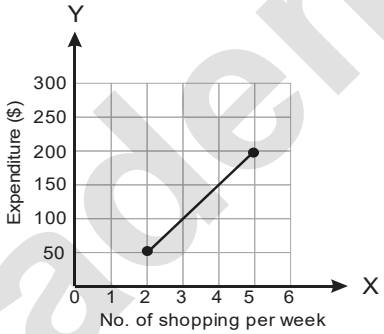
23.7

2 Find the rate of increase of % score for the following graph.



_____

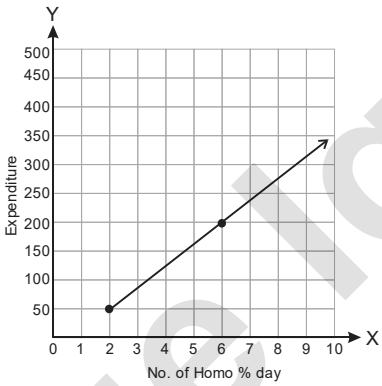
3 Find the rate of expenditure.



_____

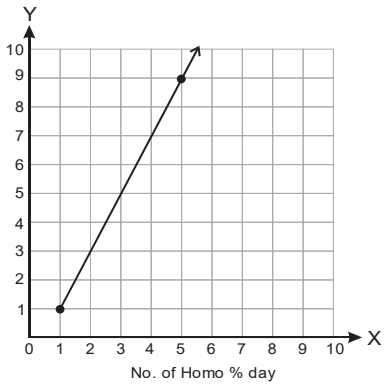
Find the rate of change '*y*' for following graph with respect to *x*.

4



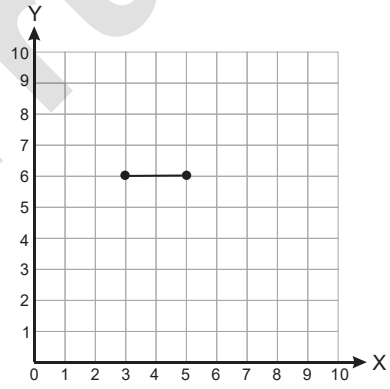
_____

5



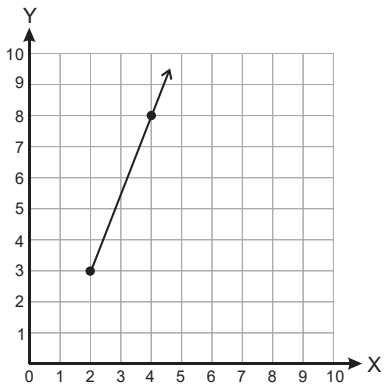
_____

6



_____

7



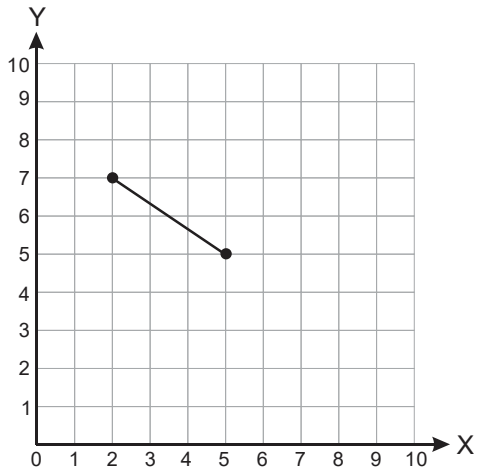
_____

23.7

Name:_____

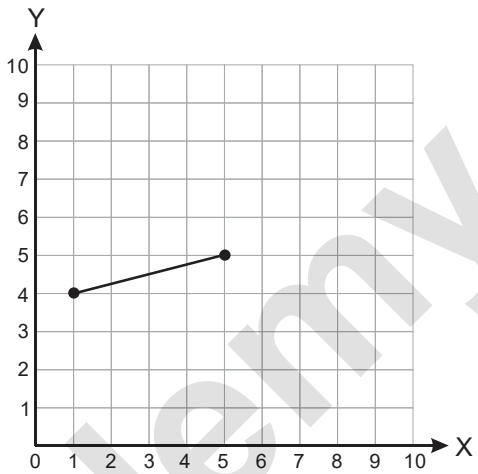
Grade:_____

8



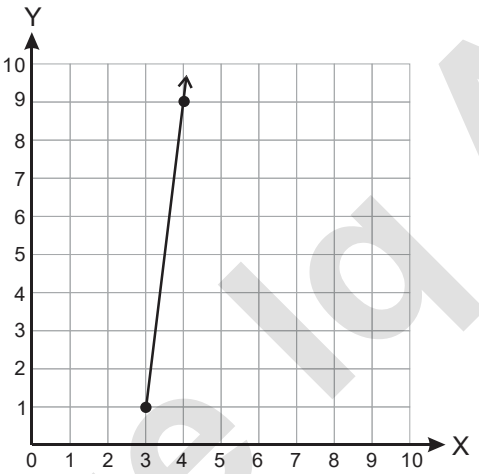
_____

9



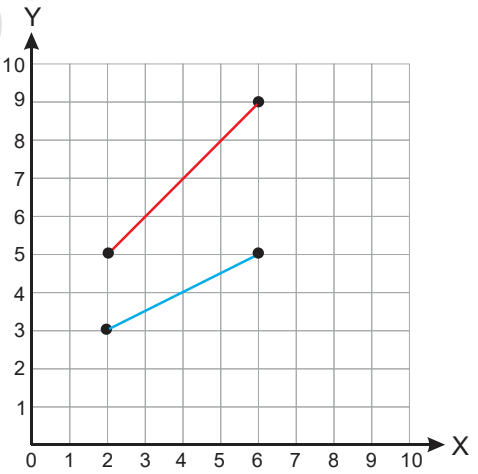
_____

10



_____

11



Which line is having greater rate of change ?

(a) Red Line ☐

(b) Blue Line ☐

Name:_____

Grade:_____

Evaluate a Linear Function

Evaluation of a linear function f(x) (or y) can be done by putting the given value of x in the expression of f(x) (given function). The value of the function is the resulting value of y or f(x).

Example Evaluate given function for given value of x.

(i) $y = 7x - 3$ at $x = 2$

(ii) $y = 2x + \frac{3}{2}$ at $x = -3$

(i) Value of $y = 7x - 3$ at $x = 2$ is

y (2) or f(2) = 7(2) - 3 = 14 - 3 = 11

(ii) Value of $y = 2x + \frac{3}{2}$ at $x = -3$ is

$$y(-3) = 2(-3) + \frac{3}{2}$$

$$= -6 + \frac{3}{2} = \frac{3}{2} = \frac{-9}{2}$$

Work sheet 23.8

Use following functions and evaluate value of functions at given conditions of x.

1. $y = 2x + 3, x = 2$

2. $y = 4x - 5, x = 3$

3. $y = 10x - 70, x = 4$

4. $y = \frac{3}{4}x - 7, x = 8$

5. $y = \frac{4x - 15}{7}, x = 4$

6. $y = \frac{17x + 9}{20}, x = -2$

7. $4y = 5x + 2, x = -3$

8. $10y = \frac{3}{4}x + 5 ; \ x = 0$

9. $\frac{7}{4}y = 2x - 15 ; x = 7$

10. $2.5y = 5x - 10 ; \ x = -3$

23.9

Name:_____

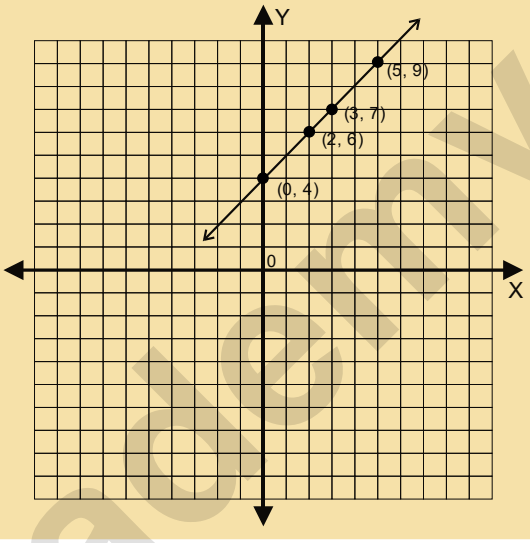
Grade:_____

Complete table for linear function

By evaluating the function for various values of 'x', different values of y can be found and can be used to

Example: Mark is collecting coins and he always collect coins in such a way that number of pennies (coins) is 4 more then dimes (coins). Thus using equation $p = d + 4$, we can complete the table and plot the graph, where 'p' is number of pennies and 'd' is number of dimes.

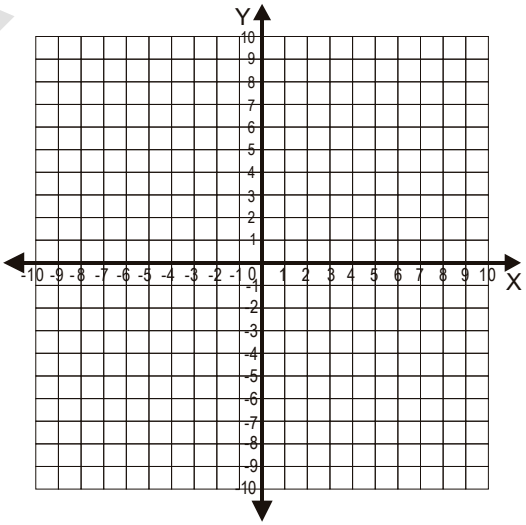
P	4	6	7	9
d	0	2	3	5



Work sheet 23.9

1. The number of orange candies is always 5 more than the number of chocolates possessed by Mathew. Complete the table using equation $O = C + 5$ and plot the graph.

C	O
1	6
3	
6	

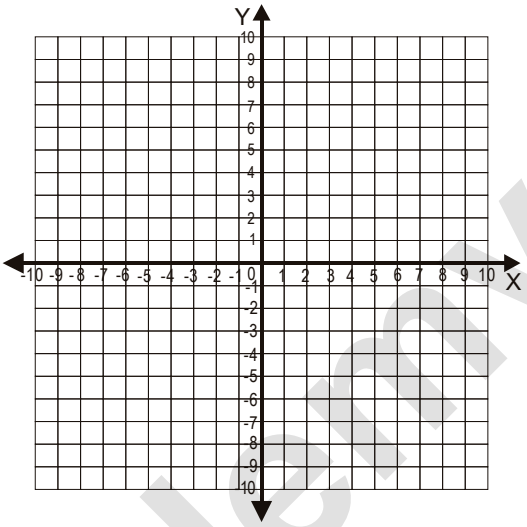


2. On her birthday Jenifer distributed candies in the class such that she gave each student 3 candies and she kept 5 candies separate for the teacher.
The total number of candies required is expressed as $C = 3S + 5$.

S	C
10	35
15	50
5	
25	

3 Suzan bought the amount of rice flour twice the amount of wheat flour .Complete the table using the given data $R = 2W$ and plot the graph.

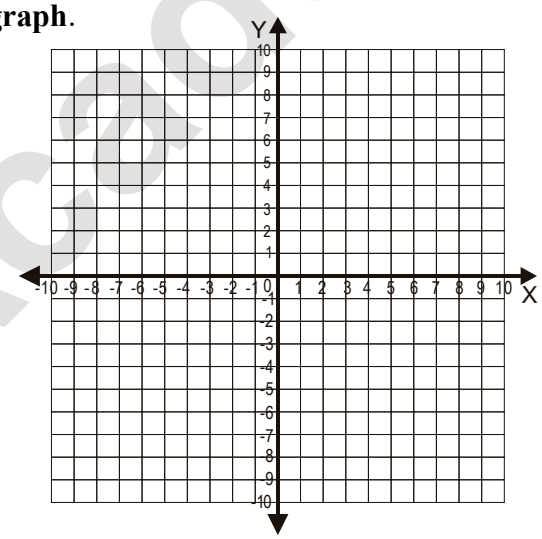
W	R
3	6
4	8
5	10
2	4



Complete the table using the given function and plot the graph.

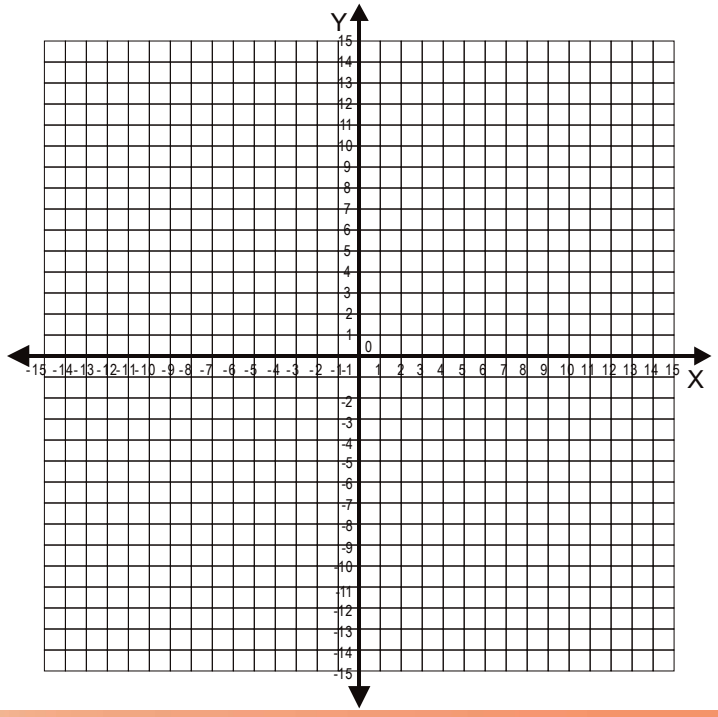
4 $y = 3x - 7$

X	1	2	3	5	7
Y	-4			8	



5 $y = 2x + 3$

X	1	-1	0	5	6
Y	5			13	15



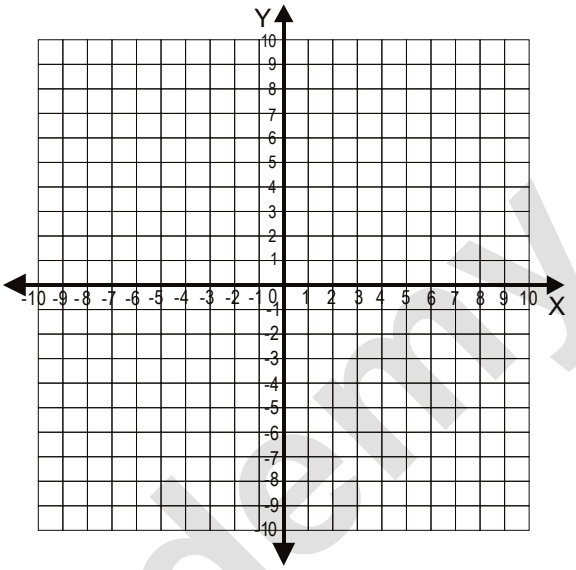
23.9

Name:_____

Grade:_____

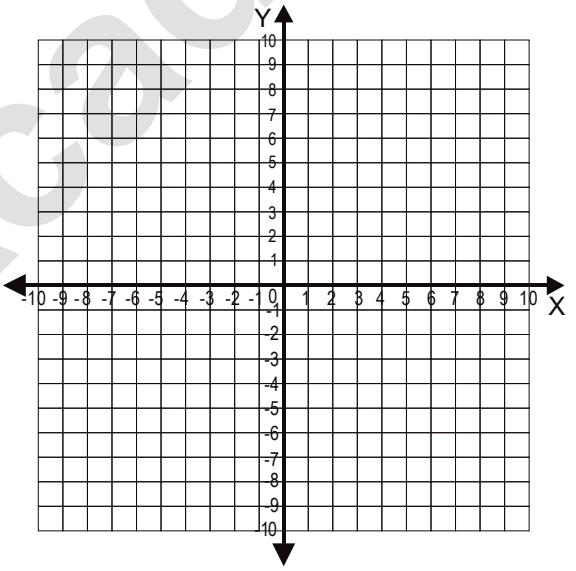
6 $y = -4x + 10$

x	0	1	3	4
y		6		-6



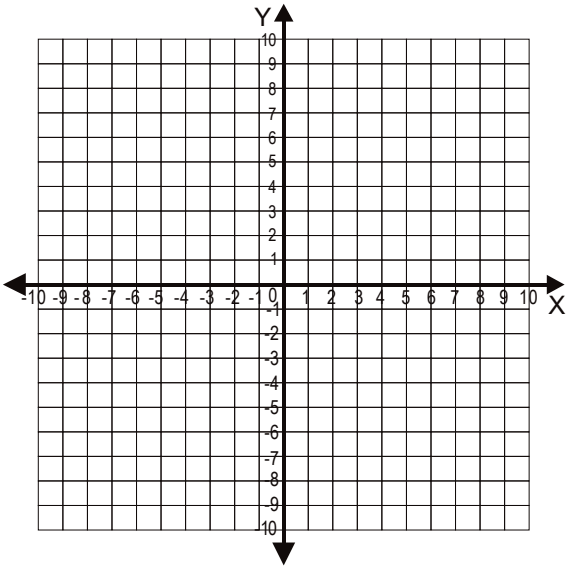
7 $y = \frac{x+2}{3}$

x	-8	-5	4	7
y	-2		2	



8 $y = \frac{5x+1}{2}$

x	0	1	3	-1
y	$\frac{1}{2}$		8	-2



Name:_____

Grade:_____

23.10

Writing linear functions from table

Example Which of the equation gives the rule for given table

x	*y*
0	1
3	7
-1	-1
5	11

(a) $y = 2x - 1$ ☐ (b) $y = 2x + 1$ ☐

(c) $y = x + 1$ ☐ (d) $y = 4x - 1$ ☐

For option (b), by putting value of $x = 0$ we get $y = 1$ and by putting value of $x = 3$ we get $y = 7$. Hence (b) option is the correct answer

Work sheet 23.10

Select the equation that gives the values for the given table.

1

x	y
0	-5
2	9
1	2

(a) y = 6x - 4 ☐

(b) y = 7x - 5 ☐

(c) y = 7x + 1 ☐

2

x	y
-2	-4
0	2
2	8

(a) y = 2x + 3 ☐

(b) y = x + 5 ☐

(c) y = 3x + 2 ☐

3

x	y
-4	-1
-2	1
0	3

(a) y = x + 3 ☐

(b) y = 2x + 1 ☐

(c) y = 3x - 1 ☐

4

x	y
-1	-1/2
2	4
4	7

(a) y = 1/2x + 3/2 ☐

(b) y = x + 3/2 ☐

(c) y = 3/2x + 1 ☐

23.10

Name:_____

Grade:_____

5

x	y
-1	-6
0	-1
1	4

(a) $y = 5x$ ☐

(b) $y = 4x + 1$ ☐

(c) $y = 5x - 1$ ☐

6

x	y
1	-1
2	-4
0	2

(a) $y = 3x - 2$ ☐

(b) $y = -4x + 3$ ☐

(c) $y = -3x + 2$ ☐

7

x	y
0	-2
2	2
-2	-6

(a) $y = 3x - 2$ ☐

(b) $y = 2x - 2$ ☐

(c) $y = 3x - 4$ ☐

8

x	y
4	9
0	-5
-2	-12

(a) $y = \frac{7}{2}x - 5$ ☐

(b) $y = \frac{3}{2}x - 5$ ☐

(c) $y = 5x - 2$ ☐

9

x	y
0	1
1	0
-1	2

(a) $3x + y = 3$ ☐

(b) $2x + y = 1$ ☐

(c) $x + y = 1$ ☐

10

x	y
0	-2
1	-1
2	0

(a) $x - 2y = 4$ ☐

(b) $x - y = 2$ ☐

(c) $2x - y = 5$ ☐

11

x	y
3	0
0	2
$\frac{-3}{2}$	3

(a) $3x + 2y = 6$ ☐

(b) $2x + 3y = 6$ ☐

(c) $x + 2y = 4$ ☐

12

x	y
1	-2
2	$\frac{1}{2}$
3	3

(a) $5x - 3y = 11$ ☐

(b) $5x - 2y = 9$ ☐

(c) $3x - 5y = 11$ ☐

Name:_____

Grade:_____

23.11

Linear functions: Word problems

Example Daisy get $3 for each packet she delivers and she gets $10 for each day as fixed payment. Write down the equation for one day salary of Daisy.

Consider 'S' be Daisy's one day salary.
As Daisy gets $3. for each packet she delivers therefore let 'p' be be number of packets.
Hence the equation will be :
$S = 3p + 10$

Work sheet 23.11

1. While working at a store, Dan earns $15 per hour. If '*y*' is the total money earned in *x* days. Then write an equation to show their relation.

_____

2. John's birthday costs $3 for every guest he invites. If the number of guest invited are x then find the total cost '*y*'.

_____

3. Roxy reads 3 books a week as part of her circular activity. Write equation showing relation between the total books she read (say y) in *x* weeks.

_____

4. Mary's birthday picnic costs $3 for every students who attends the picnic. If '*x*' students comes at picnic, then write an equation to show total cost *y*.

_____

5. George charges $5 for every pudding he sells. He has already earned $ 20. Write the equation showing the earning of the day he will make in total ? .

_____

6. Stuart earns $10 for every hour he spends in the workshop of automobile . He gets $ 20 for each day as fixed share . Write the equation for the earning of Stuart for a day ?

_____

23.12

Name:_____

Grade:_____

Identify linear and non linear functions (From graphs and equation)

We know that linear function can be written with a constant slope. If we plot linear function on a graph it will be always a straight line.

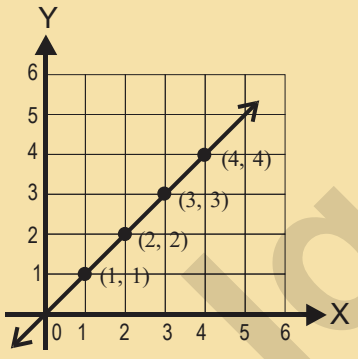
Non linear function cannot be written in this form and its graph will never be a straight line.

Example

Linear function

$y = x$

x	0	1	2	3	4
y	0	1	2	3	4



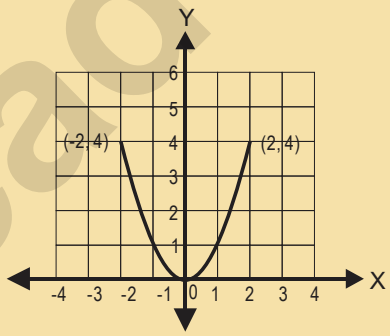
Clearly $y = x$ is of the form $y = mx + b$.
where $m = 1$ and $b = 0$

Example

Non linear function

$y = x^2$

x	0	1	-1	2	-2
y	0	1	1	4	4



Clearly $y = x^2$ is not of the form $y = mx + b$ as power of x cannot be 2 in a linear function

Work sheet 23.12

1. Is the function $y = 5x - 9$ Linear or non linear ? Linear ☐ Non Linear ☐

2. $y = -8x$ is non linear function True ☐ False ☐

3. $y = -2x^5 + 1$ is linear function True ☐ False ☐

Name:_____

Grade:_____

23.12

4. $y = \frac{-9}{2}x^3$ is non-linear function. True ☐ False ☐

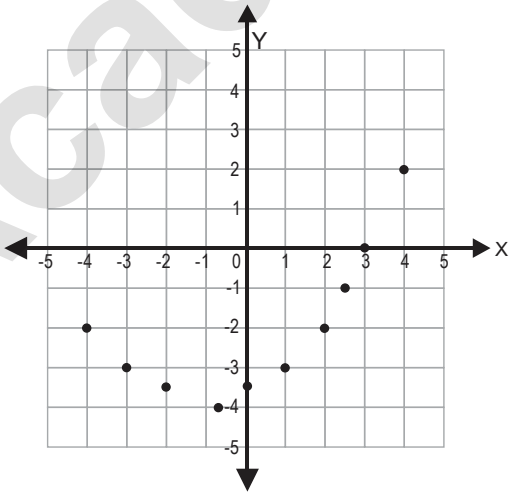
5. Is the function $y = 5^x - 5$ is linear or non linear ? Linear ☐ Non Linear ☐

6. Is the function $y = \frac{8}{x} + 1$ linear or non linear ? Linear ☐ Non Linear ☐

7. Is the function $y = -\frac{8}{9}x$ linear or non linear ? Linear ☐ Non Linear ☐

8. Is the graph shows of a linear function or non linear function ?

Linear ☐ Non Linear ☐



9. Is the function $y = \frac{4}{7}\sqrt{x + \frac{7}{3}}$ a linear or non linear ? Linear ☐ Non Linear ☐

10. Is the function $y = \frac{3}{2}x + \frac{1}{2}$ a linear or non linear ? Linear ☐ Non Linear ☐

11. Is the function $y = 7x^3 - \frac{1}{3}$ a linear or non linear ? Linear ☐ Non Linear ☐

23.13

Name:_____

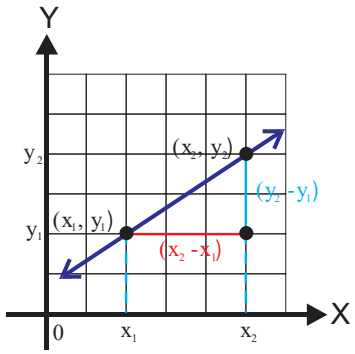
Grade:_____

Identify linear and non-linear function (From Tables)

We know that a linear function has constant rate of change.

A function's rate of change between two points A (x_1, y_1) and B (x_2, y_2) is :

$$\frac{y_2 - y_1}{x_2 - x_1}$$



A function has a constant rate of change is when its rate of change remains same between any two points. A non linear function does not have a constant rate of change.

Example The given table shows a function.

x	y	
11	15	Row I
12	12	Row II
13	5	Row III

To determine whether the function is linear or non linear, see whether it has a constant rate of change.

Pick any two points from the table and calculate the rate of change between them. The first two rows are good start.

Call the values in the first row x_1, y_1 and the values in the second row x_2, y_2.

Rate of change

$$= \frac{y_2 - y_1}{x_1 - x_1} = \frac{12 - 15}{12 - 11} = \frac{-3}{1} = -3$$

Now pick any other two rows say Row 2 and 3 . Call the values in second row x_1 and y_1 and values in the third row as x_2 and y_2

Rate of Change

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 12}{13 - 12} = \frac{-7}{1} = -7$$

The rate of change is not the same for each pair of points. So, the function does not have a constant rate of change.

Therefore, function is non linear.

Work sheet 23.13

The following table shows a function. Is the function linear or non linear ?

1

x	6	7	5
y	16	12	5

Linear ☐ Non Linear ☐

Name:_____

Grade:_____

23.13

2

x	7	8	9
y	12	15	18

Linear ☐ Non Linear ☐

3

x	12	13	14
y	21	14	7

Linear ☐ Non Linear ☐

4

x	9	12	15
y	7	10	18

Linear ☐ Non Linear ☐

5

x	8	10	12
y	5	11	16

Linear ☐ Non Linear ☐

6

x	-5	5	21
y	3	11	19

Linear ☐ Non Linear ☐

7

x	-1	3	7
y	31	39	47

Linear ☐ Non Linear ☐

8

x	2	3	4
y	7	12	19

Linear ☐ Non Linear ☐

9

x	2	6	10
y	$\frac{3}{2}$	$\frac{13}{2}$	24

Linear ☐ Non Linear ☐

10

x	-1	0	1
y	-5	-4	-3

Linear ☐ Non Linear ☐

23.14

Name:_____

Grade:_____

Satisfy a non-linear function using ordered pair

Example Does the point (–1, –6) satisfy the equation $y = x^2 + -2x - 9$?

A point is a solution to an equation if plugging in *x* and *y* results in a a true statement.
In the ordered pairr (–1,–6), –1 is the *x* - value and $y = -6$ is the *y* – value.

Put $x = -1$ and $y = -6$ into the equation.
$y = x^2 + -2x - 9$.
$-6 = (-1)^2 + (-2)(-1) - 9$
$-6 = 1 + 2 - 9$
$-6 = 3 - 9$
$-6 = -6$, Yes $-6 = -6$, so the point (–1, –6) satisfies the equation
$y = x^2 + -2x - 9$.

Work sheet 23.14

1. Ordered pair (0,–1) satisfies the equation $y = 8x^2 - 9x - 1$. True ☐ False ☐
2. Does the point (9–3) satisfy the equation $y = x^2$? Yes ☐ No ☐
3. Does (0, 1) make the equation $y = 5x^2 - 2x + 1$ true? Yes ☐ No ☐
4. Is (–8, –7) a solution to the equation $y = x^2 - 9x - 1$ true? Yes ☐ No ☐
5. Does (1, 1) satisfy the non linear equation $y = x^2$? True ☐ False ☐
6. Check whether the point (0, –2) satisfy the non linear function $y = 14x^2 - 11x - 2$? Yes ☐ No ☐
7. Does (1, –4) makes the equation $y = 2x^2 - 5x$ true ? Yes ☐ No ☐
8. Does (–2, 12) makes the equation true $y = 3x^2$? Yes ☐ No ☐
9. Does (2, 3) satisfies the given equation true $y = x^2 - 5x + 6$? Yes ☐ No ☐
10. Does (0,20) satisfy the given equation $y = 5x^2 - 20$? Yes ☐ No ☐

Name:_____

Grade:_____

Evaluation: Non-linear functions and their values

To evaluate the value of a function f(x) then for a given value of x, we simply plug in the values of x in the expression $f(x)$ and find its value.

Example

Using function rule find $f(-2)$

$$f(x) = 3x^2 - 5$$

f(-2) can be found out by plugging value of -2 in the expression f(x)

$$\therefore \quad f(x) = 3x^2 - 5$$

$$3(-2)^2 - 5 = 3(4) - 5$$

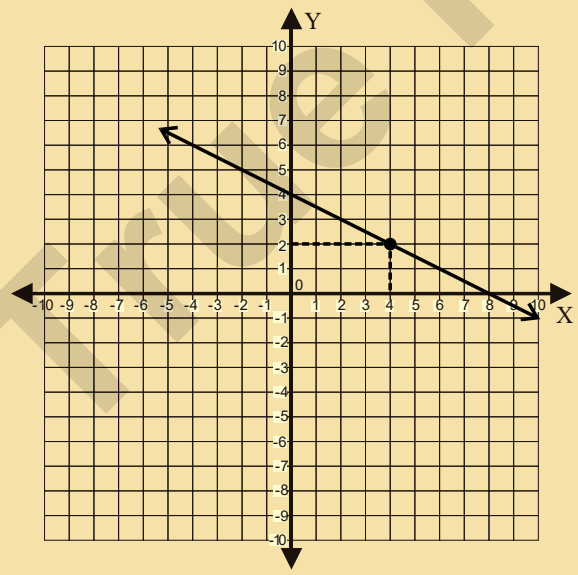
$$= 15 - 5$$

$$\therefore \quad f(-2) = 7$$

Example

Plot point on this function with x-value of 4.

Write down y -value of this point.



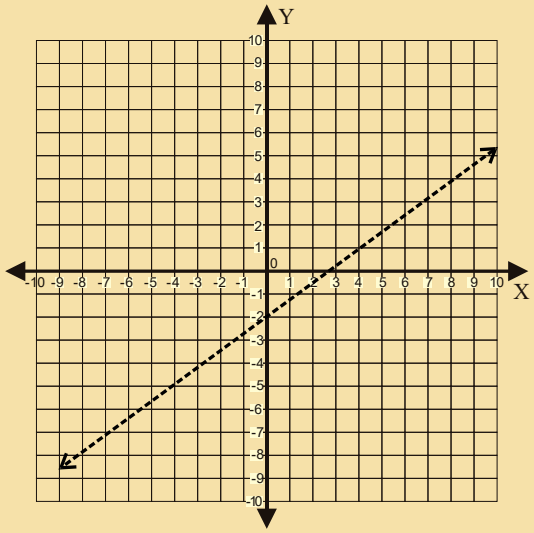
Here were see that after plotting value of x = 4 for the graph we see that corresponding value for y for that point or graph is 2

Hence y-value is 2.

Example

Use the garph of the function to complete the table.

Input	Output
0	-2
2	
4	1
6	



Input	Output
0	-2
2	0
4	1
6	3

23.15

Name:_____

Grade:_____

Work sheet 23.15

1. Use the following function rule to find $f(-9)$.
$f(x) = x^2 - 3x$

_____

2. If a rule function $f(x) = 6|x| - 7$
then $f(-5)$.

_____

3. If $f(x) = (-7 + x)^2$, then what will be f(10).

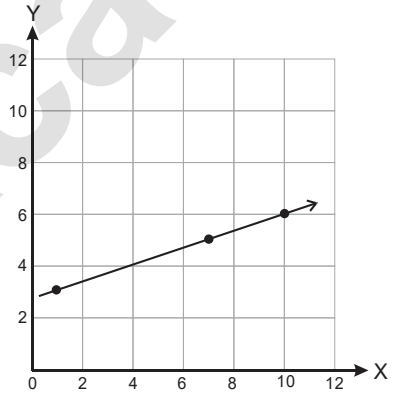
_____

4. Use the following function rule to find f (- 3).
$f(x) = -7|x|$

_____

5. Use the graph of the function to complete the table (insert graph diagram)
(insert table)

Input	Output
1	3
7	5
10	

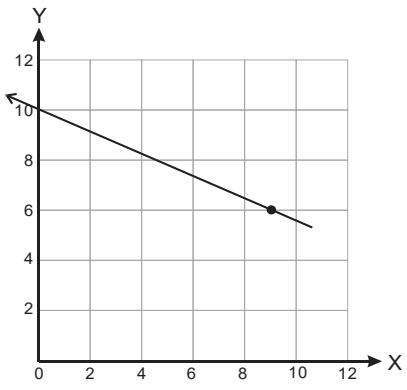


6. If $f(x) = 3|9 - x|$ is a non-linear function, then find its value at $x = 9$.

_____

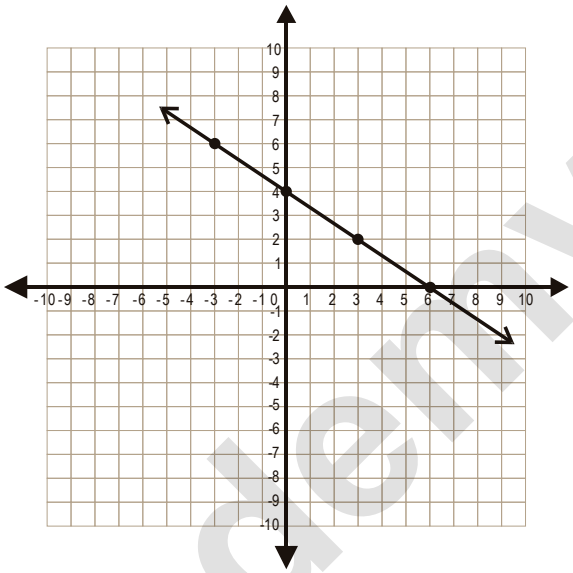
7. Plot the point on this function with x value of 9.
(Insert graph diagram)
What is the value of this point?

_____



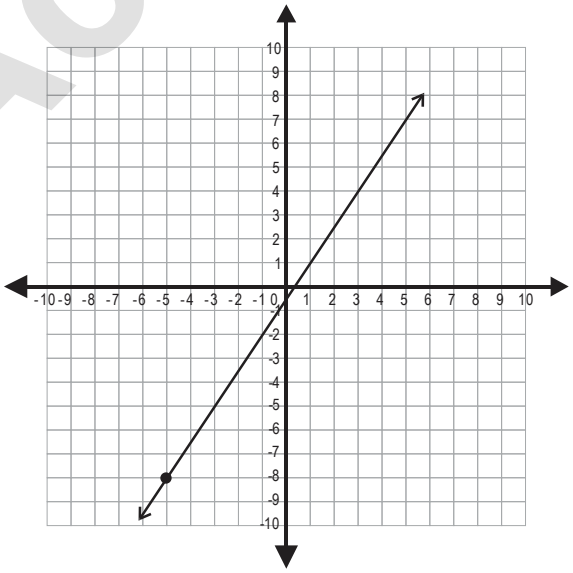
8 Use the graph of the function to complete the table.

x	y
-3	
0	4
3	
6	0
9	



9 Use the graph of the function to complete the table.

x	y
-5	
-3	-5
1	
4	



23.16

Name:_____

Grade:_____

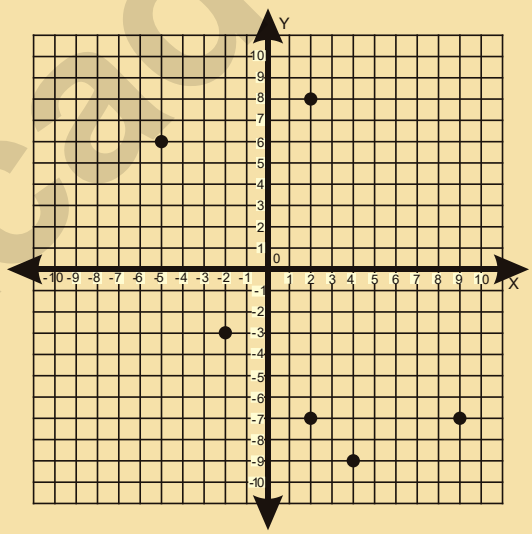
Domain and Range of Functions

The domain of the function includes all the possible values that can be input in a function or all x coordinates taken in brackets. For example domain of $f(x) = x^2$, is all real numbers . Also the domain of $g(x) = \frac{1}{x}$, is all real number except $x = 0$.

The range of the function includes all the possible outputs a function can generate or in other words the set of all y - coordinates taken in brackets.

Example If the following coordinates are given for the function, (-12, 3), (-2, 9)
(-10, 6), (3, -8)
Write the domain and range of the function.
Here the domain of the function is the input value or the x-values, while the range of the function is the y-values or the output values of the function.
Hence, domain = { -12, -10, -2, 3}
and range = { -8, 3, 6, 9}

Example
In the following graph
(i) What is the domain of the function
(ii) What is the range of the function.
(i) Here the domain of the function given is all the values of x - it can take (or all input value)
Hence, domain = (-5, -2, 2, 4, 9)
(ii) The range of the function is the output values (or the y - values) and hence range = {-9, -7, -3, 6, 8)



Work sheet 23.16

Write down the range of the following functions separated by comas in { }.

1. (-5,4), (2, -7), (3,-9), (6,6)
   Range = { _____ }

2. (3, 2), (4, 8), (9, 1), (2, -5)
   Range = { _____ }

3. (1,1), (2, 4), (-3, 9), (-1, 1)
   Range = { _____ }

4. (-8, 2), (-5, 5), (1, 9), (2, 7)
   Range = {_____}

5 $\left(\frac{-3}{2},8\right),\left(-9,\frac{7}{11}\right),(14,12),\left(\frac{13}{4},16\right)$

Range = { _____}

6 $(2,7),(-11,0)(-1,-3),(6,-6)$

Range = { _____}

Write down the domain of the following functions.

7 (3,5), (2, -7), (-6, 1), (-7, 0)

domain = { _____}

8 (1, -10), (-3, -9), (1, 1), (0, 3)

domain = {_____}

9 (1, 5) (-1, 6), (2,7) (0,0)

domain = { _____}

10 (5,1) (-6, 0), (-3, -1), (2, -2)

domain = {_____}

11 $\left(\frac{5}{2},3\right)(6,8)\left(16,\frac{-47}{5}\right)(-12,23)$

domain = { _____}

12 $(-8,19),(14,11)(0,-13)(19,-26)$

domain = {_____}

13 (1, 2) (2,4) (3,6) (4,8)

domain = { _____}

14 (3, 7) (4, 8) (5, 9) (8, 10)

domain = {_____}

23.17

Name:_____

Grade:_____

Understanding functions

While it has been discussed earlier that a relation is said to be a **function**, when for each value of x in domain the relation gives a unique value of y.

Hence its contradiction is also true, that if in a relation, for any value of x in domain if the relation gives more than one value of y, the relation is **not a function**.

It has already been discussed graphically that in a **vertical line test**, if in a graph a vertical line cuts the graph more than once, at any point(x-value), then that graph is not a function.

Work sheet 23.17

1. The table shows a relation that is a function. Select a value from the option so that the relation remains a function.

(a) (-5, -3) ☐ (c) (3, -1) ☐

(b) (4, 6) ☐ (d) (-7, 7) ☐

x	y
4	-3
-7	8
-5	-1
0	5

2. Complete the table so the relation is not a function.

x	y
	0
7	4
2	2
0	-3

3. Which among the following relation is a function?

x	y
-2	1
0	4
2	1
4	8

a) ☐

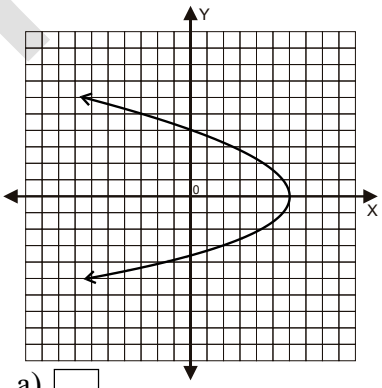
x	y
0	-5
1	-2
2	1
0	3

b) ☐

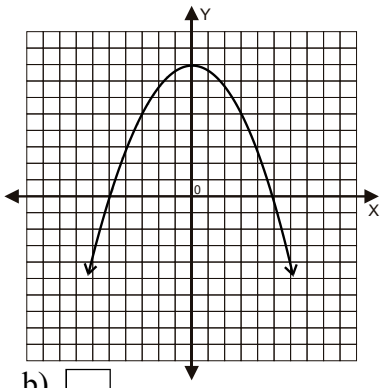
x	y
-5	4
-3	-4
0	0
2	5

c) ☐

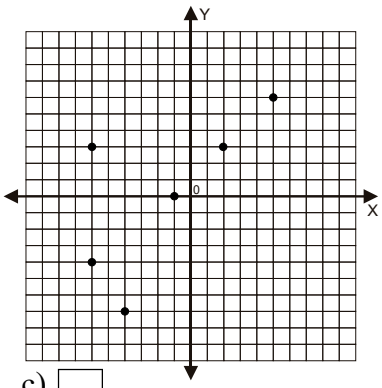
4.



a) ☐



b) ☐



c) ☐

5 Which these given relation(s) is a function?

x	y
-2	2
-1	0
0	4
3	6

a) ☐

x	y
0	7
-1	3
0	-2
3	0

b) ☐

x	y
-2	3
-1	5
0	3
3	7

c) ☐

6 Which these given relation(s) is a function?

x	y
1	1
3	4
5	10
1	-1

a) ☐

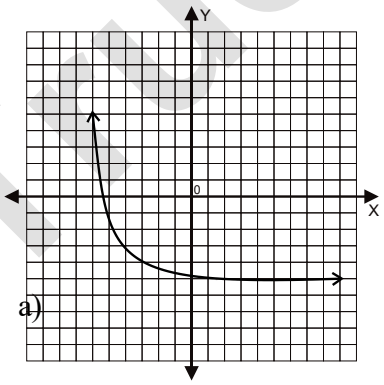
x	y
1	5
3	7
5	12
7	7

b) ☐

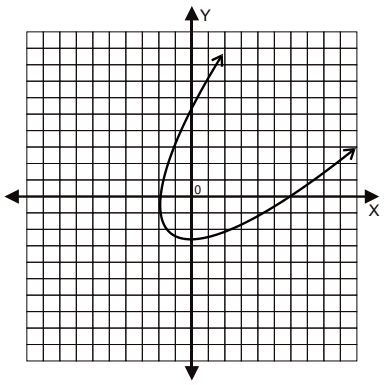
x	y
1	-2
3	-1
5	0
3	1

c) ☐

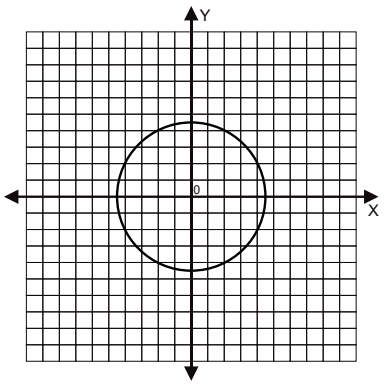
7 Which these given relation(s) is a function?



a) ☐



b) ☐



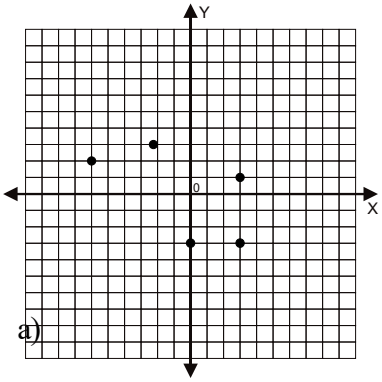
c) ☐

23.17

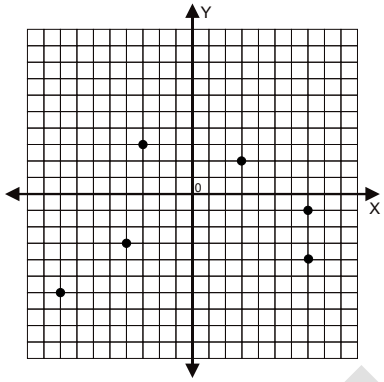
Name:_____

Grade:_____

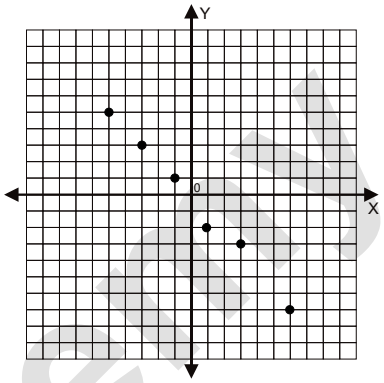
8 Which these given relation(s) is a function?



a) ☐



b) ☐



c) ☐

9 Which these given relation(s) is a function?

x	y
-5	7
-3	5
0	3
0	-1

a) ☐

x	y
-5	5
-3	3
-1	-2
-3	7

b) ☐

x	y
-5	-10
-3	-2
-1	4
0	-2

c) ☐

10 Input such values to fill the table to make the following relation not a function.

x	y
-3	8
-2	5
-1	10
1	15

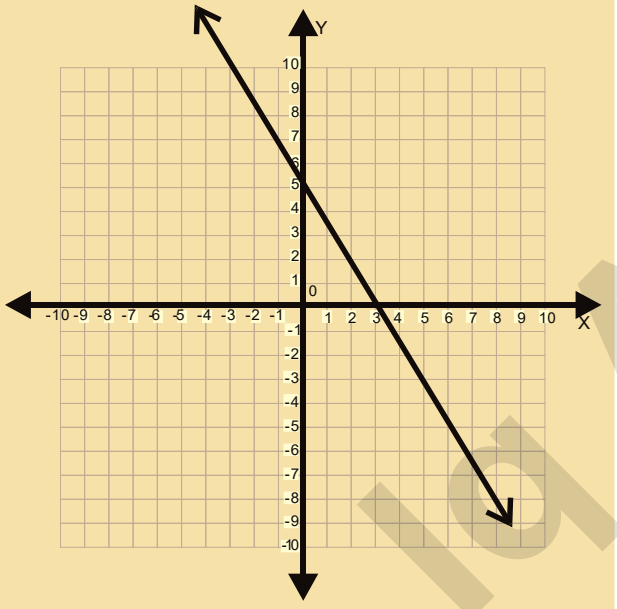
Name:_____

Grade:_____

Comparing functions

Functions can be compared on the basis of their slope and intercepts.

Example Function- A is given as y = 4/3 x + 3, and Function-B is given to pass through points (0, 5/2) and (-5/4, 0) while the Function-C is given as the following graph. Find which function is having the greatest slope and which function is having greatest y- intercept?



Graph for Function C

We can see that the Function -B has slope of 2 and the equation as

(y-5/2)=2(x-0) which be comes

y = 2x + 5/2

For function C slope is, (5-0)/(0-3) = -5/3

And we can see y- intercept = 5

Now we can clearly state that function B has the greatest slope among the functions.

While function C has the greatest y intercept among the functions.

Work sheet 23.18

1. Function A has equation, y = 3x + 2 and function B is a linear function including points (0,3) and (3.5, 0).
Select the correct option.

(a) y - intercept of function A is more than that of function B. ☐

(b) y - intercept of function B is more than that of function A ☐

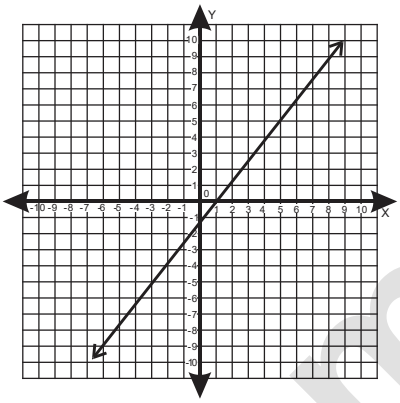
(c) Both functions A and B are having same y - intercepts. ☐

Name:_____

Grade:_____

2 Function A

x	y
2	8
-1	-4
4	16



Function B

Write down a function C using integral values only so that the range of change is between the rate of change of both functions.

$y = (\_\_\_\_\_)\,x + (\_\_\_\_\_)$

3 Function A is a linear function given in table

x	0	3	5
y	2	-3	-10

Function B is also a linear function with y-intercept $\frac{3}{2}$ and x-intercept of 5.

4 Function A has equation $y = 5x + 1$ and function B is a liner function passing through points $(-2, -6)$ and $(2, 10)$ select the correct option:

a) y-intercept of function A is more than that of function B ☐

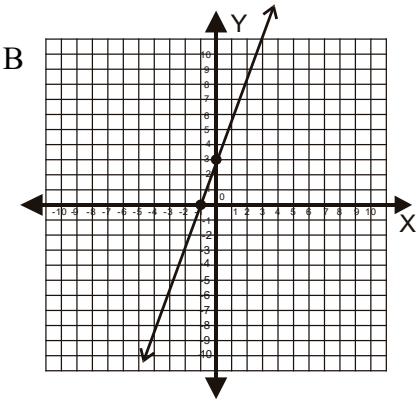
b) y-intercept of function B is more than that of function A ☐

c) Both functions A and B are having same y-intercept. ☐

5 Function A

x	y
-1	-3
1	7
2	12

Function B



Write down a function C using integral values for fill ups so that rate of change is between both the functions A and B

$y = \_\_\_\_\_\,x + 2$

6 Taking the same function A and function B of question 6, write down a function D, such that the y-intercept of function is between that of both the functions?

$y = 7x +$ _____

7 Function A is a linear function given in table.

x	-3	-1	1
y	-8	0	8

Function B is also a linear function with y-intercept 5 and x-intercept of -2.
Select the correct option by comparing functions

	Greater for A	Greater for B	Same for A & B
Slope			
y-intercept			

8 Function A is a linear function given in table.

x	-2	-8	-4	-6
y	4	16	12	8

Function B is $y = 3x + 8$. Which function has more slope and which function home greater y-intercept.

23.19

Name:_____

Grade:_____

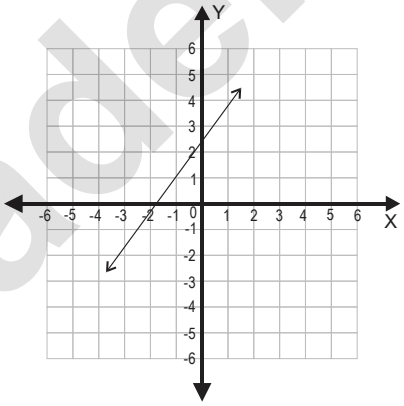
Linear and Non-Linear functions

A function is said to be a **linear** function, if the slope (or the rate of change of y with respect to x) remains constant.

A function is **non-linear** if its slope through out the graph is not constant (or changes).

Work sheet 23.19

1 Does the following graph shows a linear function ?



(a) yes, because the slope is constant ☐

(b) no, because slope is positive ☐

(c) no, because the y intercept is positive ☐

(d) yes because x - intercept is negative ☐

2 Which equation is a linear among the following?

(a) $y = x^2 + 3x$ ☐

(b) $y = \frac{2x^2}{2} - 5$ ☐

(c) $y = \frac{5}{2}x - 1$ ☐

(d) $y = \sqrt{x} + 7$ ☐

3 Which of the following shows a linear function ?

(a) (-2, -5), (4, 10), (1, 2-5) ☐

(b) (-2, 3), (1, 5), (1, -1) ☐

(c) (1, 8), (4, 12), (4, 17) ☐

(d) (0, 3), (4, 0), (2, 5) ☐

4 Select the linear function from the following

(a) $y = \frac{8}{15}\sqrt{x^3}$ ☐

(b) $y = \frac{2x^2}{5} - 10$ ☐

(c) $y = 7z + 5$ ☐

Name:_____

Grade:_____

23.19

Mention the functions as linear or non-linear function.

5 $y = 7x^2 + 3x + 2$

_____

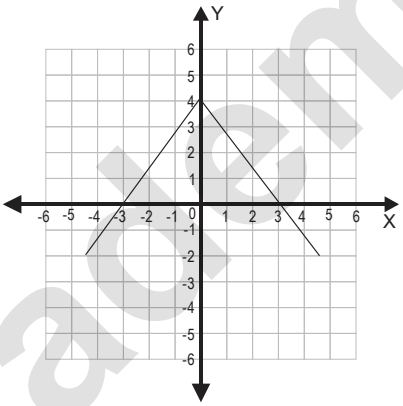
6 $y = 41 - 17x^2$

_____

7 $y = \frac{2}{3} - \frac{5}{2}x$

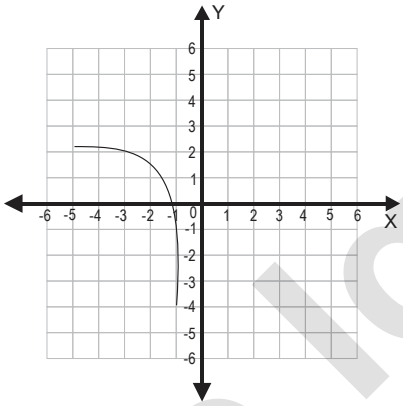
_____

8



_____

9

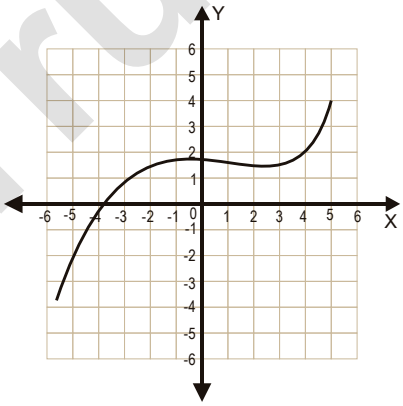


10

x	y
3	2
-6	-4
-1.5	1

_____

11



_____

12

x	y
2	4
4	6
6	8
8	10

_____

23.20

Name:_____

Grade:_____

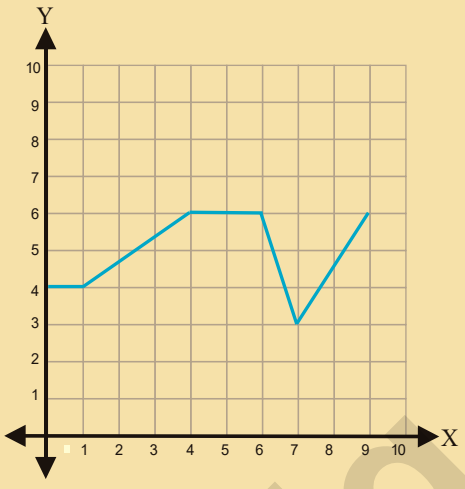
Sketch and describe graphs

Graphs can be analysed from its slope.

If the slope is *positive* then the rate of change is **increasing**. If the slope is negative then the rate of change is **decreasing**.

If the slope is *zero*, there is **no rate of change** (rate of change = 0).

Example The graphs shown below. Select which options are correct from the following given options:



a) The functions is constant between $x = 0$ and $x = 3$

b) The functions is increasing between $x = 3$ and $x = 6$

c) The functions is increasing between $x = 6$ and $x = 8$

d) The functions is increasing between $x = 2$ and $x = 4$

For selecting the correct option it is required that the correct span of the function is identified having constant slope (decreasing or increasing).

The graph of the function is having zero slope between $x = 0$ and $x = 1$, and between $x = 4$ and $x = 6$.

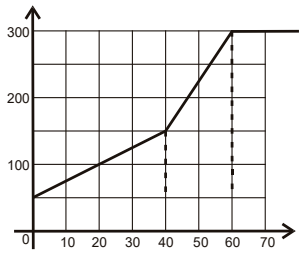
The graph is increasing between x=1 and $x = 4$, and between $x = 7$ and $x = 9$.

The graph is decreasing between $x = 6$ and $x = 7$.

Therefore it can be clearly seen that option (d) is the correct option and rest of the options are false.

Work sheet 23.20

1 Harry hiked to local mountain and in the interval he took energy boosting drink. The graph shows the elevation graph for given interval of time.



Check if the statements are true of false

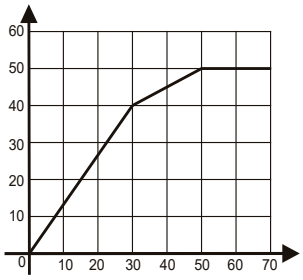
	True	False
Harry's elevation was decreasing from minute 0 to minute 40.	☐	☐
Harry's elevation was increasing from minute 40 to minute 60.	☐	☐
Harry's elevation was constant from minute 60 to minute 70.	☐	☐

Name:_____

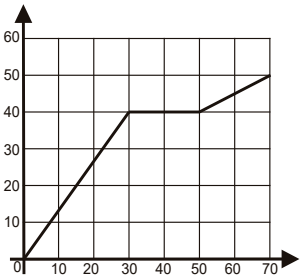
Grade:_____

23.20

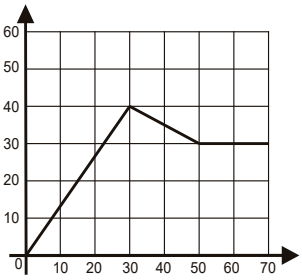
2 Martha is an athelete and went to her friend's home by running. At first she was running faster but after 30 minutes she started running slower and further after 45 minutes she stopped to have her snacks. which graph represent the distance martha ran.



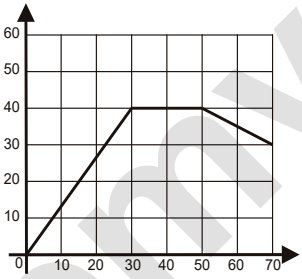
(a) ☐



(b) ☐



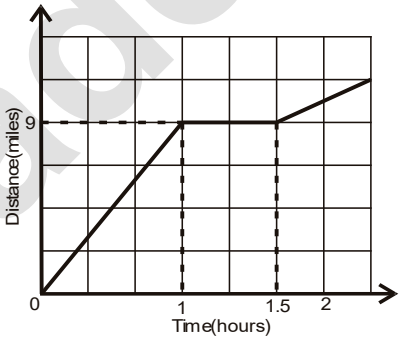
(c) ☐



(d) ☐

3 Here the graph shows the distance travelled by Zenny in miles , in respect to the hours covered.

Chose all the statements that describe the graph.



(a) Zenny travelled for 9 miles and then travelled at slower speed for 30 minutes and finally come to halt. ☐

(b) Zenny travelled 9 miles in 60 minutes and then stopped for 30 minutes and then finally again travelled at a slower rate ☐

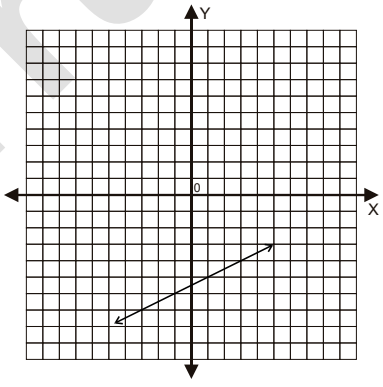
(c) Zenny travelled for 60 minutes and after 60 minutes travelled at a higher speed for 30 minutes and eventually travelled at slower speed. ☐

(d) Zenny travelled for 90 minutes at a uniform speed and after that she travelled at a slower speed. ☐

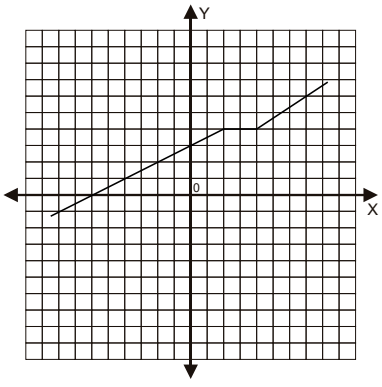
Classify following graphs as

" Linear and increasing " Linear and decreasing" or "Non - linear graphs"

4



5



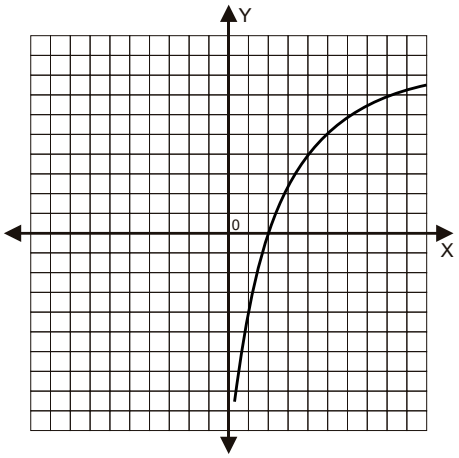
_____

23.20

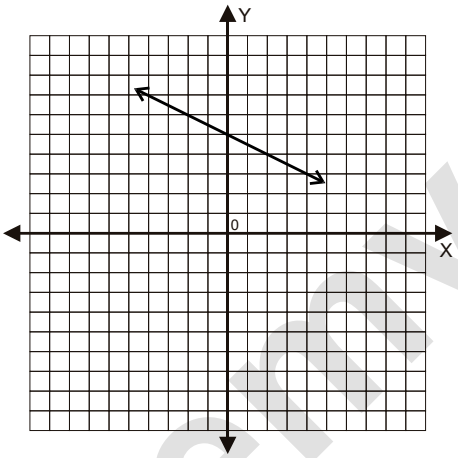
Name:_____

Grade:_____

6



7

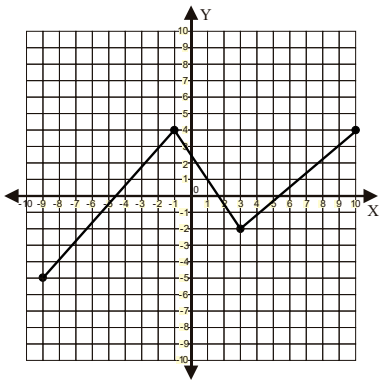


8 Following graph shows a function. Select the statement that describe about the graph (i).

(a) The function is constant between x = -5 and x = -2 ☐

(b) The function is decreasing between x = -3 and x = 2 ☐

(c) The function is increasing between x = 4 and x = 10 ☐



Graph (i)

9 For which interval the function is increasing. Select the option that apply to graph (i)

(a) $-5 < x < 0$

(b) $-1 < x < 3$

(c) $-10 < x < -1$

(d) $1 < x < 11$

(e) $3 < x < 10$

10 Select the statement that is true for function of graph (i)

(a) The slope of function is constant for $-9 < x < -2$ ☐

(b) The domain of the function is between -2 and 4 ☐

(c) The domain of the function is between -10 and 11 ☐

(d) The slope of the function is increasing between $x = 2$ to $x = 11$ ☐

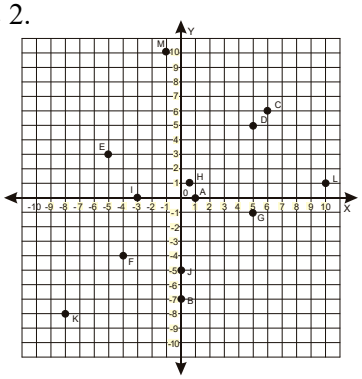
Name:_____

Grade:_____

Answers

Worksheet 23.1

1. & 2.

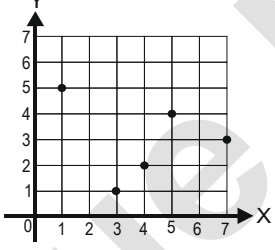


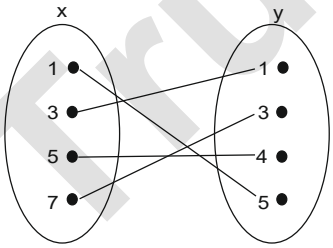
3. A (-4, 8) B(6, 6)
   C(-2, 3) D(-4, -3)
   E(6, 2) F(3, -3)
   G(-8, -3) H(3, -6)
   I(2, 9) J(-5, -8)
   K(9, -2)
4. C
5. A(-5, 2), B(3, 1)

Worksheet 23.2

1. Yes 2. (c)
3. Yes 4. Yes
5. b, c 6. d
7. (b) 8. (b)
9. (a)
10.

x	y
1	5
3	1
5	4
7	3
4	2





Yes

11. Domain: (68 70, 125)
    Range: (2, 3)
    False
12. Domain: (7, 8, 10, 13)
    Range: (-8, -2, 0)
    No
13. (-2, 1), (0, -3), (0, 3) (1, 0), (1, 4), (2, 1), (3, -4)
    Domain: -2, 0, 1, 2, 3
    Range: -4, -3, 0, 1, 3
    No
14. Domain: (1, 3)
    Range: (2, 4)
    Yes

Worksheet 23.3

1. Yes 2. No 3. No
4. Yes 5. (a) 6. (a)
7. (b) 8. (d)

Worksheet 23.4

1. No 2. No 3. No
4. No 5. No 6. Yes
7. No 8. No 9. Yes
10. Non-linear 11. No
12. Yes 13. No 14. No
15. Yes 16. Yes

Worksheet 23.5

1. Dependent variable = vacation days.
   Independent variable = Months worked
2. Dependent variable - chance of scoring marks
   Independent variable - study
3. Dependent variable - fare
   Independent variable - miles travelled
4. Dependent variable - eyes getting tired.
   Independent variable - watching television
5. Dependent variable-glass for thirst quench
6. Dependent variable -scores
   Independent variable-rehearsal
7. Dependent variable - extra bonus
   Independent variable overtime worked.
8. Dependent variable - electricity supply
   Independent variable -sun hours in a day
9. Dependent - Playing
   independent -legs gets weak

Worksheet 23.6

1. $-\frac{1}{4}$ *or* -0.25
2. -16
3. $\frac{13}{15}$ *or* 2.6
4. 1
5. 1
6. 1
7. $\frac{6}{5} = 1.20$
8. 0
9. A
10. C
11. 1
12. $0.5 = \frac{1}{2}$
13. $\frac{4}{3} = 1.33$
14. -5
15. (c)
16. (a)
17. 1
18. $\frac{3}{2} = 1.5$
19. (c) 20. (b)
21. 10/8 22. 30/12

Worksheet 23.7

1. 12.5
2. 263.88
3. 50
4. 37.5
5. 2

Name:_____

Grade:_____

6. 0
7. 2.5
8. $-0.66 = \frac{-2}{3}$
9. $0.25 = \frac{1}{4}$
10. (a)
11. 8

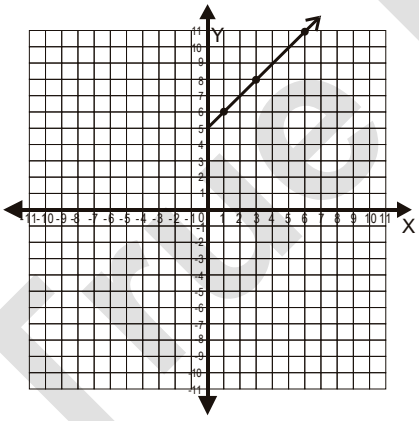
Worksheet 23.8

1. 7
2. 7
3. -30
4. -1
5. $\frac{1}{7}$
6. -1.25
7. -3.25
8. $\frac{1}{2} = 0.5$
9. $\frac{-4}{7}$
10. -10

Worksheet 23.9

1.

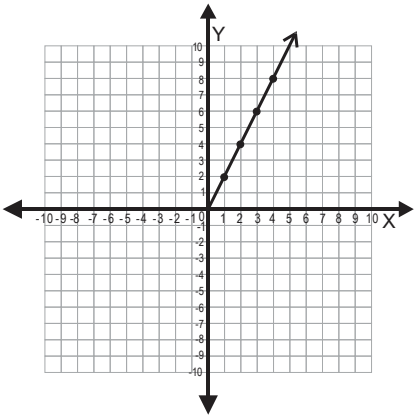
C	O
3	8
6	11



2.

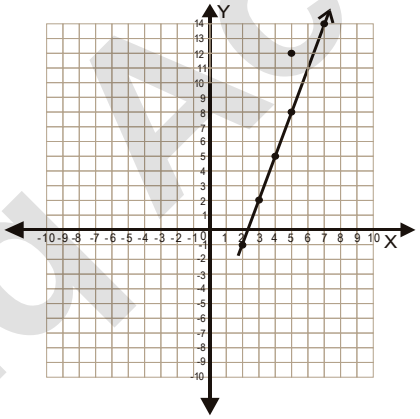
S	C
5	20
25	80

3.



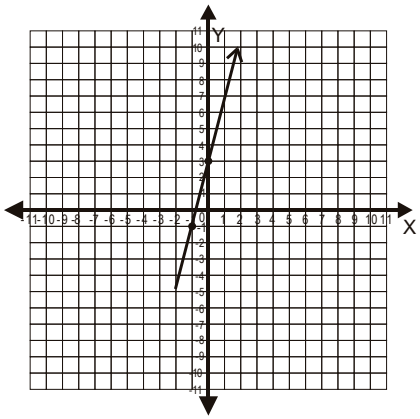
4.

x	y
2	-1
3	2
7	14



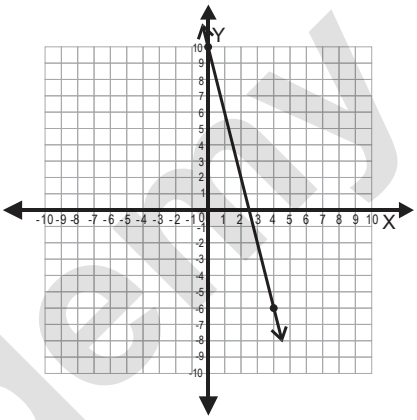
5.

x	y
-1	1
0	3



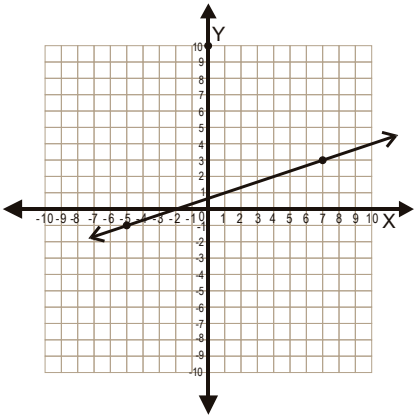
6.

x	y
0	10
3	-2



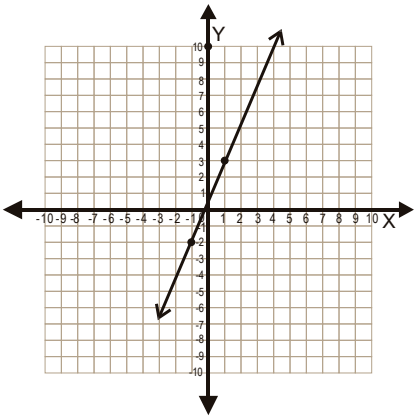
7.

x	y
-5	-1
7	3



8.

x	y
1	3
-1	-2



Name:_____

Grade:_____

Worksheet 23.10

1. (b)
2. (c)
3. (a)
4. (c)
5. (c)
6. (c)
7. (b)
8. (a)
9. (c)
10. (b)
11. (b)
12. (b)

Worksheet 23.11

1. $y = 15x$
2. $y = 3x$
3. $y = 3x$
4. $y = 6x$
5. $y = 5x + 20$
6. $y = 10x + 20$

Worksheet 23.12

1. Linear
2. False
3. False
4. False
5. Non linear
6. Non linear
7. Linear
8. Non-linear
9. Non linear
10. Linear
11. Non linear

Worksheet 23.13

1. Non linear
2. Linear
3. Linear
4. Non linear
5. Non linear
6. Non linear
7. Linear
8. Non-linear
9. Non-linear
10. Linear

Worksheet 23.14

1. True
2. No
3. True
4. No
5. True
6. Yes
7. No
8. Yes
9. No
10. No

Worksheet 23.15

1. 108
2. 23
3. 9
4. -21
5. 6
6. 0
7. (9, 6)
8.

x	y
-3	6
0	4
3	2
6	0
9	-2

9.

x	y
-5	-8
-3	-5
1	1
4	5

Worksheet 23.16

1. Range = {4, -7, -9, 6}
2. Range = {1, 2, 8, -3}
3. Range = {1, 4, 9}
4. Range = {2, 5, 7, 9}
5. Range = $\left\{\frac{7}{11}, 8, 12, 16\right\}$
6. Range = {-6, -3, 0, 7}
7. Domain = {-7, -6, 2, 3}
8. Domain = {-3, 0, 1}
9. Domain = {-1, 0, 1, 2}
10. Domain = {-6, -3, 2, 5}
11. Domain = $\left\{\frac{5}{2}, 6, -12, 16\right\}$
12. Domain = {-8, 0, 14, 19}
13. Domain = {1, 2, 3, 4}
14. Domain = {3, 4, 5, 8}

Worksheet 23.17

1. (c)
2. Either 2 or 7 or 0
3. a, c
4. b
5. b
6. b
7. a
8. b
9. c
10. -2, 3

Worksheet 23.18

1. (c)
2. m = 2 or 3, $b \in I$
3. Slope > B & y-intercept > for A
4. (b)
5. $\frac{1}{2}$
6. $\frac{5}{2}$
7. Slope > for A
   Y-intercept > for B
8. Slope > B, y-intercept > B

Worksheet 23.19

1. (a)
2. (c)
3. (a)
4. (c)
5. Non-linear
6. Non-linear
7. Linear
8. Non-linear
9. Non-linear
10. Non-linear
11. Non-linear
12. Linear

Worksheet 23.20

1. (a) False (b) True
   (c) True
2. (a)
3. (b)
4. Linear
5. Non-linear
6. Non-linear
7. Linear
8. (c)
9. (e)
10. (d)